Bridging the gap to A-Level

Introduction

Congratulations on choosing to study AS Level Maths. To help you prepare, this booklet will enable you to brush up on some of the skills you have learned at GCSE. You are going to need to use them from day 1, and if you don't have a good grasp of the basics you need to work on them NOW so that you can start with confidence.

> **Do the questions in this booklet in pencil, then check your answers. If you get something wrong, revise the topic then try again. The aim is to get EVERYTHING right!**

Studying AS Maths is about learning how to solve problems, and getting stuck is part of the learning process. You should expect to get stuck while working through this booklet but these are all GCSE techniques that you will need to master.

There are loads of great resources on the internet to help you, but if you get stuck we recommend **ExamSolutions.net** which contains video tutorials for all GCSE Higher content. We also recommend using this site throughout the A Level course.

> http://www.examsolutions.net/maths-revision/syllabuses/GCSE/period- 1/Higher/module.php
>
>

> **There will be a TEST in your first week at college based on the material in this booklet. Please bring the completed booklet with you to your first maths lesson.**

If you want a more comprehensive revision guide, there are several books available from Amazon but please note it is NOT compulsory to buy either of these:

> Collins Maths - Bridging GCSE and A Level: Student Book
> Head Start to AS Maths by CGP Books

Finally, there is an interactive online course called 'Step up! To A Level Maths' hosted by the University of Plymouth that is packed with extra resources and examples:

http://www.cimt.plymouth.ac.uk/projects/mepres/step-up/default.htm

GCSE Maths

If you're naturally good at maths you can do well without much extra studying

It's the answer that matters most, but you should show working

You have an exercise book to keep all your work together

Nobody minds how you set out your workings so long as you get there in the end
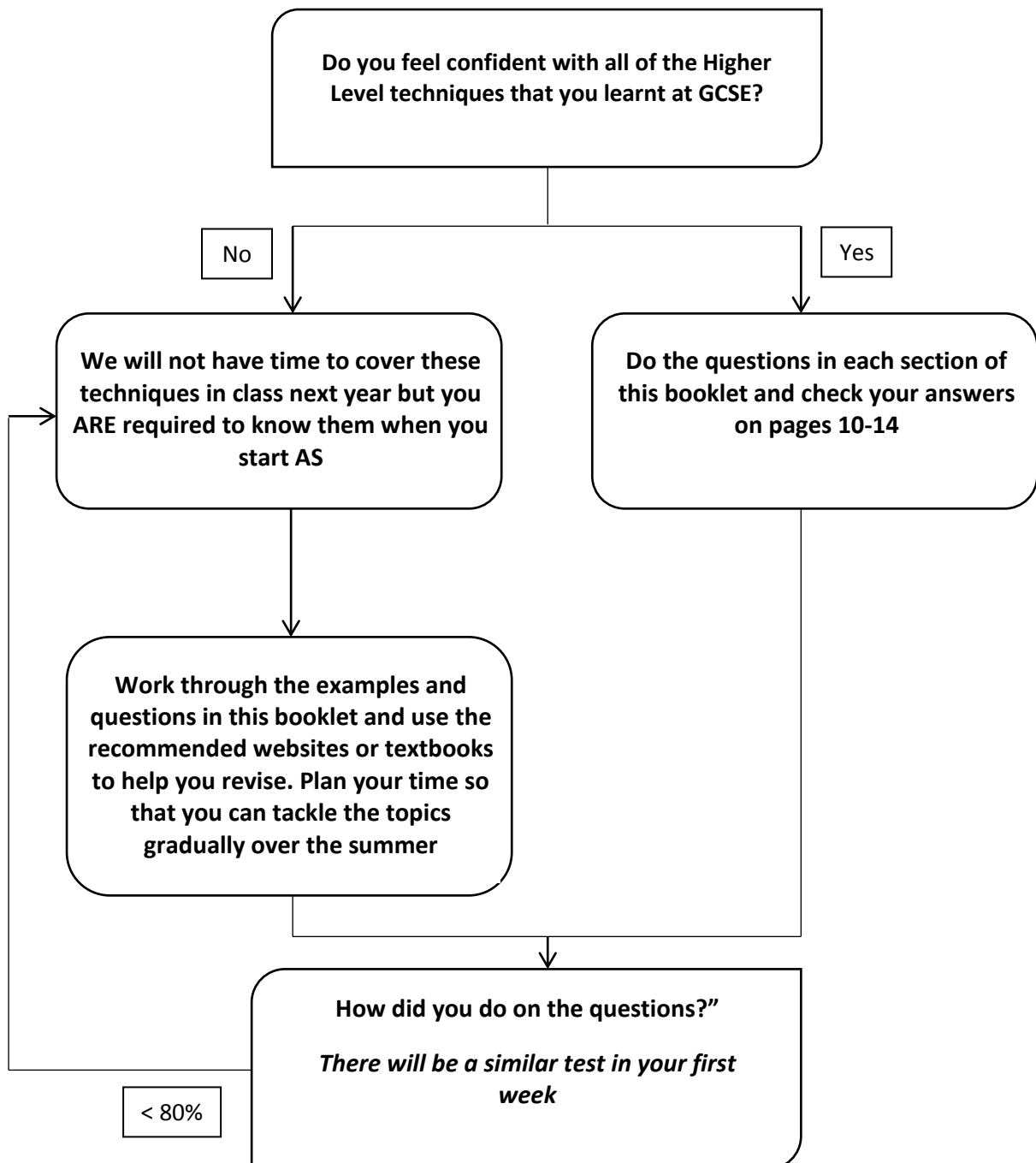
AS Level Maths

You will do a lot of study outside of class.

It's the method that matters, not the answer. Often you are given the answer but need to show steps in the method.

You will need to keep neat, accurate and well ordered notes.

How you present your work can make a big difference to whether you get the right answer at all and whether anyone can understand your method.

We will not have time to cover these techniques in class next year but you ARE required to know them when you start AS Maths

Do you feel confident with all of the Higher Level techniques that you learnt at GCSE?

No

Yes

We will not have time to cover these techniques in class next year but you ARE required to know them when you start AS

Do the questions in each section of this booklet and check your answers on pages 10-14

Work through the examples and questions in this booklet and use the recommended websites or textbooks to help you revise. Plan your time so that you can tackle the topics gradually over the summer

How did you do on the questions?"

There will be a similar test in your first week

< 80%

Number

1. Which of the following are integers?

   3   -2.8   0.4 $\frac{3}{4}$ 7.92   -9   202   0

2. Which of the following values are rational and which are irrational?

   4.7 π $\sqrt{8}$ $\frac{1}{5}$ -7 $\sqrt{16}$ 12.452   3.1

3. Evaluate the following without using a calculator, giving your answers in their simplest form.

   $\frac{2}{9} \times \frac{3}{5}$ $\frac{1}{6} \div \frac{2}{3}$ $\frac{5}{6} + \frac{3}{5}$ $\frac{5}{7} + \frac{3}{8}$

Indices, expanding and factorising

4. Simplify the following:

   $x^5 \times x^3$ $16y^7 \div 4y$ w^0 $(3m^3)^4$

5. Write 3^{-3} as a fraction.

6. Evaluate the following without using a calculator:

   $\left(\frac{2}{5}\right)^2$ $81^{0.5}$ $8^{2/3}$ $81^{-1/2}$

7. Multiply out the brackets and simplify where possible:

   $(x + 2)(x - 5)$ $(x + 3)^2$ $(3x - 2)(x + 4)$ $(x + 1)(x - 3)(x + 2)$

8. Factorise the following:

   $6x + 24$ $4x + 12xy$ $n^2 - 9$ $4x^2 - 16$ $x^2 - 7$

Surds

9. Simplify the following:

   $\sqrt{3} \times \sqrt{5}$ $(\sqrt{7})^2$ $\frac{\sqrt{49}}{\sqrt{7}}$ $\sqrt{12} + 2\sqrt{3}$ $(1 + \sqrt{5})^2$

10. Rationalise the denominators:

   $$\frac{3}{\sqrt{5}}\qquad \frac{\sqrt{2}}{3\sqrt{3}}\qquad \frac{2}{3 + \sqrt{5}}\qquad \frac{\sqrt{3}}{1 - \sqrt{2}}$$

Solving and rearranging

11. Solve these equations:

$$7x - 4 = 31 \qquad 4(x - 3) = 2(x - 1) \qquad \frac{x + 2}{3} + \frac{2x}{5} = x + 2$$

12. Make x the subject:

$$y = 4x + 5 \qquad y = \frac{2x-1}{3} \qquad y = 2x^2z - 5 \qquad y = \frac{4x - 2}{x + 1}$$

Quadratics

13. Factorise and solve:

$$x^2 + 5x + 6 = 0 \qquad x^2 - 3x - 10 = 0 \qquad 2x^2 - 3x - 2 = 0 \qquad 3x^2 - 8x = -4$$

14. Solve the following using the quadratic formula:

$$x^2 + 2x - 10 = 0 \qquad 2x^2 - 5x - 1 = 0$$

15. Solve by completing square leaving your answer as surds:

$$x^2 - 4x - 2 = 0 \qquad 2x^2 + 4x - 7 = 0$$

16. Complete the square for:

$$x^2 + 6x + 8$$

Then sketch the graph of $y = x^2 + 6x + 8$, labelling the turning point and intercepts.

Algebraic fractions, inequalities and simultaneous equations

17. Simplify:

$$\frac{16\,x^3y^3}{4\,x^2y} \qquad \frac{3x\,y^3}{(3xy)^2} \qquad \frac{x^2 - 4}{x^2 - x - 6}$$

18. Simplify:

$$\frac{8x^2}{y^2} \times \frac{3y^3}{4x} \qquad \frac{3(x + 2)^2}{12} \times \frac{4}{2x + 4} \qquad \frac{3x^2 - 21x}{x + 2} \div \frac{x(x - 7)}{9x + 18} \qquad \frac{3}{x + 1} + \frac{2x - 3}{x^2}$$

19. Solve the following:

$$8x + 3 \leq 4x \qquad 3(4 - x) > 3 \qquad 3x^2 + 2 < 14$$

$$7x^2 - 4 \geq 59 \qquad x^2 - 4x + 10 \geq 2x + 5$$

20. Draw a set of axes, show the region that satisfies the following inequalities:

$y > 3x - 2$ $y < x + 2$ $y + x > -1$

21. Solve these simultaneous equations:

$2x + y = 2$	$3x - 2y = 1$	$y = x^2 + 3$	$3y = 2(x^2 - 3)$
$x - 3y = 8$	$5x - 3y = 7$	$y - 2x = 18$	$2x - y = 2$

Proof and Functions

22. Prove that the sum of any 3 consecutive even numbers is a multiple of 6.

23. Prove that for any two numbers, the product of their difference and their sum is equal to the difference of their squares.

24. $f(x) = \frac{x+5}{3}$ and $g(x) = x - 3$

Evaluate $f(4)$ Find $fg(x)$ Find $f^{-1}(x)$

Straight line and quadratic graphs

25. What is the gradient and y-intercept of the line $2x + 3y = 4$

26. Point P has coordinates (4,3) and point Q has coordinates (-1,1).

Derive the equation of the line that passes through P and Q.
Find the length of the line segment PQ correct to 3 SF.

27. A line R has the equation $y = 3x + 2$

Find the equation of a line parallel to R which passes through (1, 2).
Find a line perpendicular to R which passes through (3, 2).

28. Sketch the graph of $y = x^2 - 7x + 10$. Label the x and y intercepts and the coordinates of the turning point.

Harder graphs and transformations of curves

29. Sketch the following graphs:

$y = x^3$ $y = \frac{1}{x}$ $y = \frac{-1}{x}$

30. This graph shows the number of rabbits (R) over time (m) where m is measured in months. The equation of the line is $R = ab^m$. Find the values of $'a'$ and $'b'$.
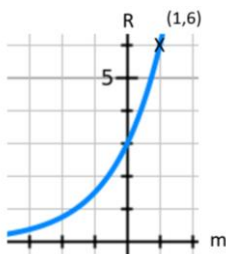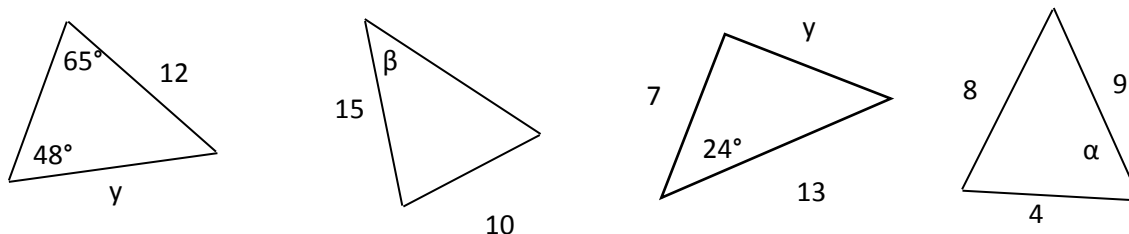

31. Find the equation of the tangent to $x^2 + y^2 = 259$ at the point (3,4).

32. Sketch the graph of $y = f(x)$ where $f(x) = x^2$. On the same axes draw the following transformations:

$y = f(x) + 2$ $\qquad$ $y = f(x + 2)$ $\qquad$ $y = -f(x)$

33. Find the unknown on each of these triangles giving your answer to 2dp (measurements in cm):
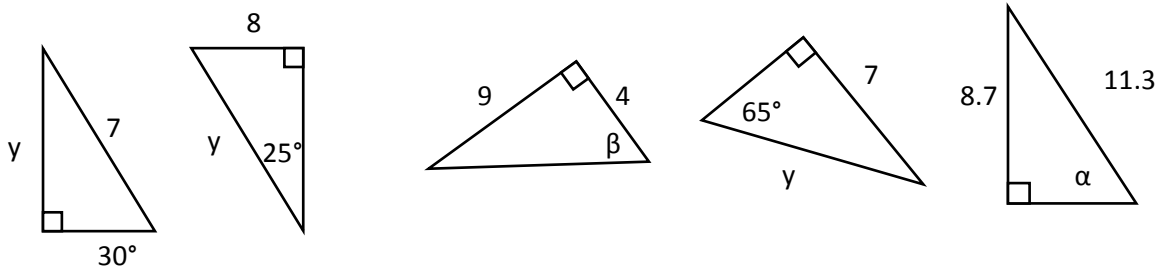
34. Find the unknown on each of these triangles giving your answer to 1dp (measurements in cm):

35. ABCD is a parallelogram: O, P, Q and R are the midpoints of the sides.

$$\overrightarrow{AB} = \boldsymbol{a} \ and \ \overrightarrow{BC} = \boldsymbol{b}$$

A   P   B
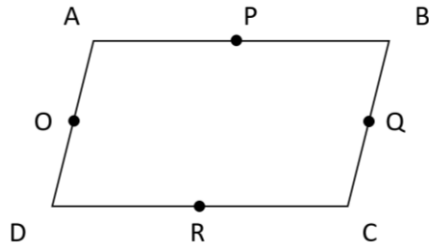
*Find the following vectors in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$:*

$$\overrightarrow{AC} \quad \overrightarrow{CQ} \quad \overrightarrow{BR} \quad \overrightarrow{QP} \quad \overrightarrow{RA} \quad \overrightarrow{PD}$$

O   Q

D   R   C

36. This diagram shows a triangle where M is the midpoint of $\overrightarrow{XZ}$ and N is the midpoint of $\overrightarrow{YZ}$. Given that: $\overrightarrow{XM} = 3\boldsymbol{a} - \boldsymbol{b}$ and $\overrightarrow{NZ} = 2(\boldsymbol{a} - \boldsymbol{b})$

Show that $\overrightarrow{XY}$ and $\overrightarrow{MN}$ are parallel.
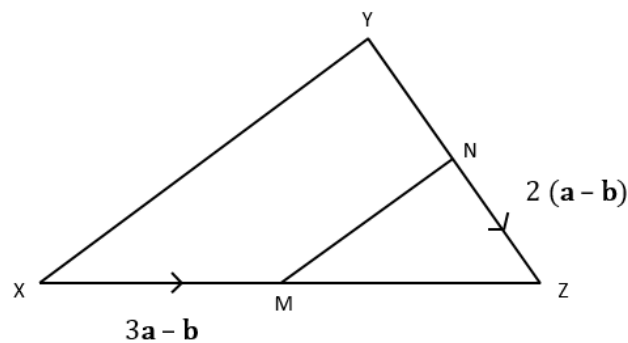
37. Describe how a random sample of size 50 can be selected from a population of 500.

38. The weights of cakes in a bakery are shown in the table and histogram on the next page:
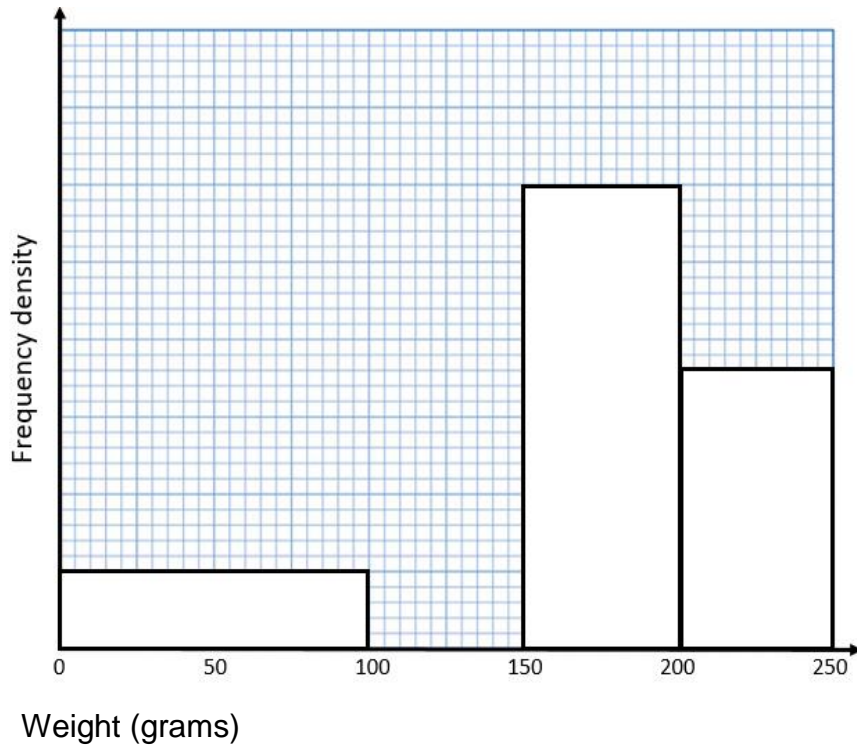
Weight (grams)

Weight 'w' in grams	Frequency
$0 < w \leq 100$	50
$100 < w \leq 150$	100
$150 < w \leq 200$	150
$200 < w \leq 250$	

Use the available information to:

Label the vertical axis

Complete the table

Add the missing bar

39. Find the mean, median, mode and range of these numbers:

5 3 -2 0 -3 2 1 1 4 2 6 11 -4
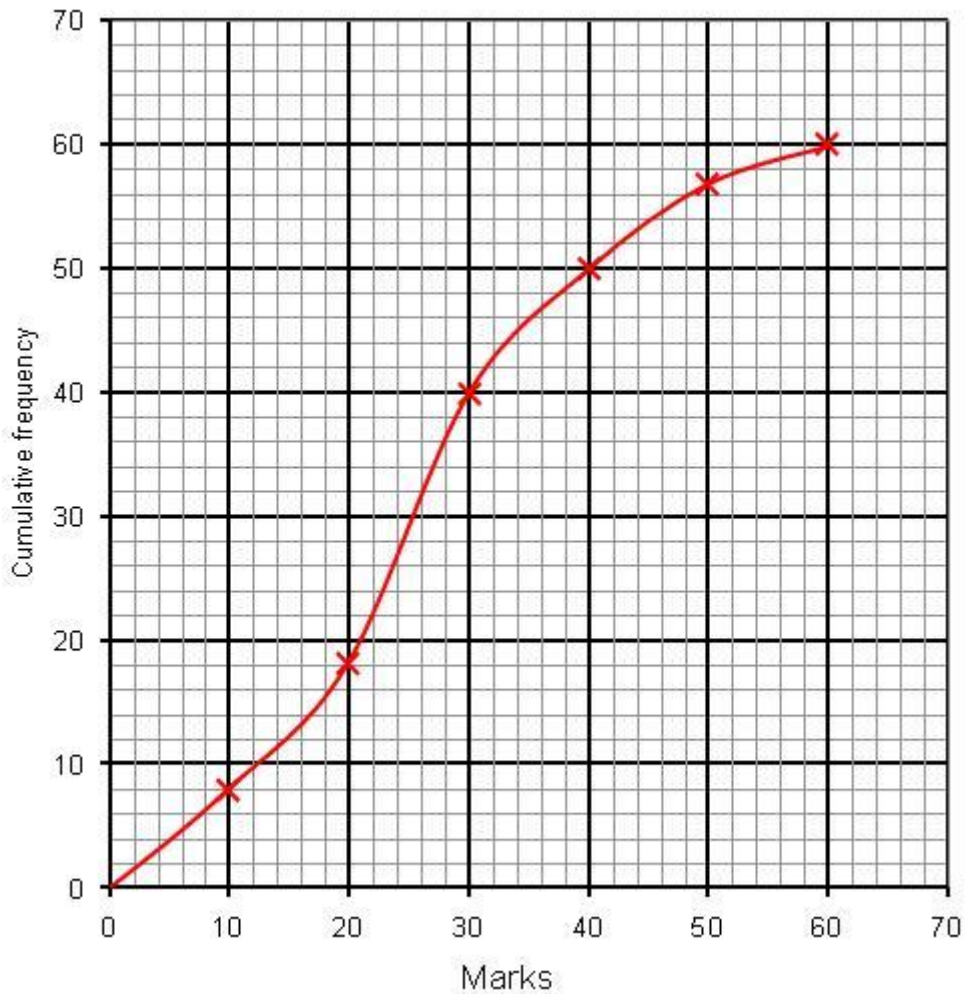
40. This table shows the length of carrots grown in the garden:

Length' L' in cm	Frequency
$0 < L \leq 10$	5
$10 < L \leq 15$	22
$15 < L \leq 20$	19
$20 < L \leq 25$	18
$25 < L \leq 30$	6

Use the available information to:

Write down the modal class

Estimate the mean

Draw a cumulative frequency curve

41. Use this cumulative frequency curve to find the median, LQ, UQ and IQR:

Probability

42. Olivia asked 70 students if they liked salt and vinegar (S) and cheese and onion crisps (C). Her results are shown in this Venn diagram:
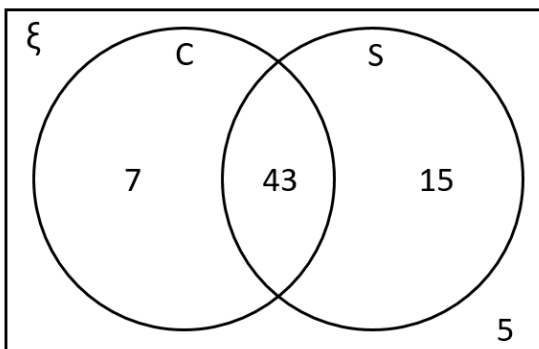
A student is selected at random, what is the probability that they:

Like cheese and onion

Like neither flavour

Like both flavours

Only like one flavour

Don't like salt and vinegar

Answers

1) 3, -9, 202, 0
2) Rational: 4.7, 1/5, -7 $\sqrt{16}$, 12.451, 3.1
   Irrational: $\sqrt{8}$, π
3) 3.2/15, ¼, 43/30 or 1 13/30, 61/56 or 1 5/56
4) $x^8, 4y^6, 1\,8m^{12}$
5) 1/27
6) 4/25, 9, 4, 1/9
7) $x^2 - 3x - 10 \qquad x^2 + 6x + 9 \qquad 3x^2 + 10x - 8 \qquad x^3 - 7x - 6$
8) $6(x + 4) \qquad 4x(1 + 3y) \qquad (n - 3)(n + 3) \qquad (2x - 4)(2x + 4) \qquad (x - 7)(x + 7)$
9) $\sqrt{15}, 7, \sqrt{7}, 4\sqrt{3}, (6 + 2\sqrt{5})$
10) $3\frac{\sqrt{5}}{5}, \frac{\sqrt{6}}{9}, \frac{3 - \sqrt{5}}{2}, -\sqrt{3}(1 + \sqrt{2})$
11) $x = 5, x = 5, x = -5$
12) $x = \frac{y-5}{4}, x = \frac{3y+1}{2}, x = \sqrt{\left[\frac{y+5}{2Z}\right]}, (y + 2), (4 - y)$
13) $x = 2 + \sqrt{6}\ or\ x = 2 - \sqrt{6}$

   $x = -5\ or\ x = -2 \qquad x = -\dfrac{1}{2}\ oe\ x = 2 \qquad x = \dfrac{2}{3}\ or\ x = 2$
14) $x = -1 \pm \sqrt{11} \qquad x = \frac{5 \pm \sqrt{33}}{4}$

15) $x = 2 \pm \sqrt{6} \qquad x = -1 \pm \frac{3}{2}\sqrt{2}$
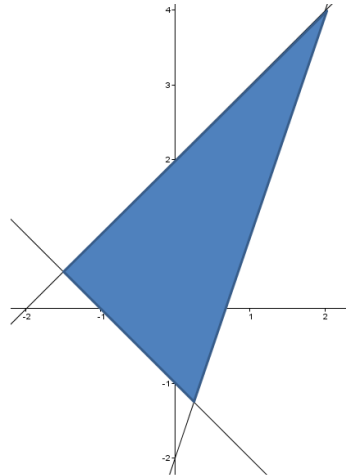
16) $(x + 3)^2 - 1$
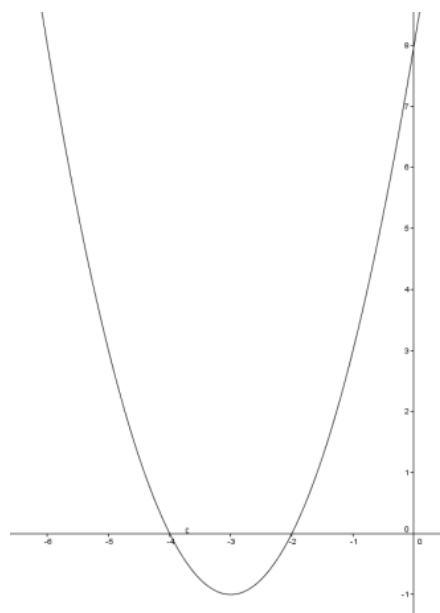
17) $4xy^2$ $\dfrac{y}{3x}$ $\dfrac{x-2}{x-3}$

18) $6xy$ $\dfrac{(x+2)}{2}$ 27 $\dfrac{5x^2-x-3}{x^2(x+1)}$

19) $x \le -\dfrac{3}{4}$ $x < 3$ $-2 < x < 2$ $x \ge 3 \ or \ x \le -3$ $x \le 1 \ or \ x \ge 5$

20)

21) $x = 2, y = -2$ $x = 11, y = 16$

    $x = -3, y = 12 \ \ or \ \ x = 5, y = 28$

    $x = 0, y = -2 \ \ or \ \ x = 3, y = 4$

22) $2n + (2n + 2) + (2n + 4) = 6n + 6 = 6(n + 1)$ $QED \ multiples \ of \ 6$

23) $(n - m)(n + m) = n^2 - m^2$

24) 3 $\dfrac{x+2}{3}$ $3x - 5$

25) $Gradient -\dfrac{2}{3}$ $y \ intercept \dfrac{4}{3}$

26) $5y - 2x = 3$ $length \ PQ = 5.39 \ (to \ 3 \ SF)$
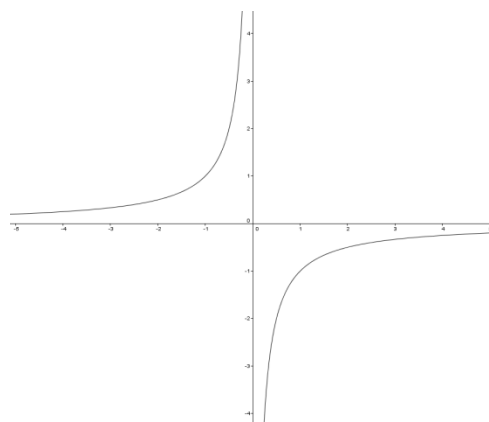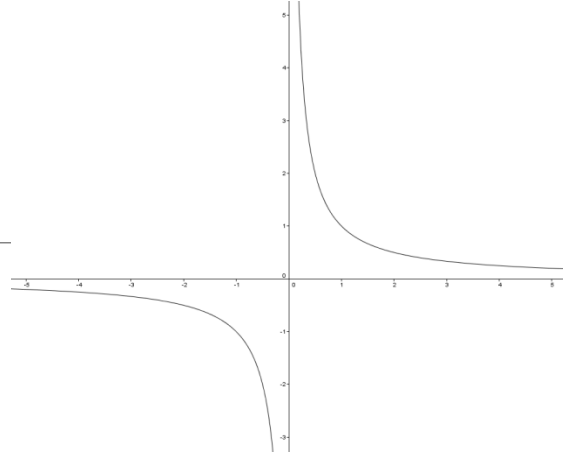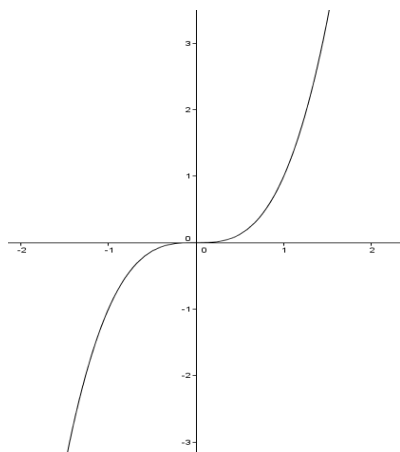
27) $y = 3x - 1$ $3y + x = 9$

28) $Graph:$
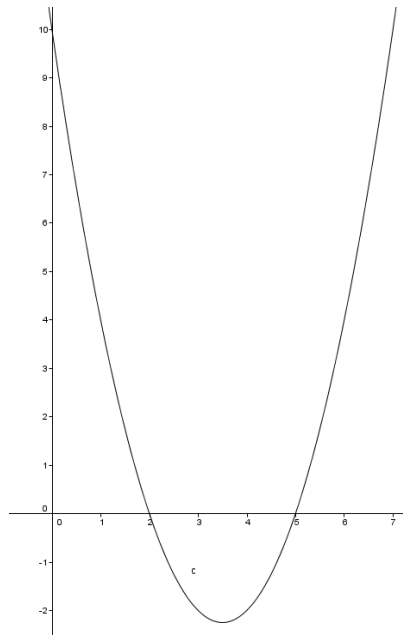
29) $y = x^3$ $y = \dfrac{1}{x}$

30) $a = 3 \quad b = 2$

31) $4y + 3x = 25$

32)
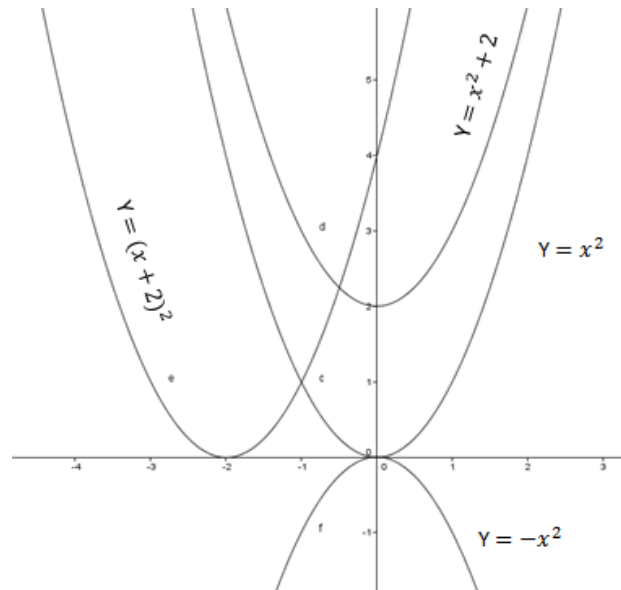
33) $3.50cm \quad 18.93cm \quad 66.04° \quad 7.72cm \quad 50.35°$

34) $14.6cm \quad 40.3° \quad 7.2cm \quad 62.7°$

35) $\boldsymbol{a + b} \qquad -\dfrac{b}{2} \qquad \boldsymbol{b - a} \qquad -\dfrac{1}{2}(\boldsymbol{a + b}) \qquad -\left(\dfrac{a}{2} + \boldsymbol{b}\right) \qquad \boldsymbol{b} - \dfrac{a}{2}$

36) $\overrightarrow{XY} = 2(3\boldsymbol{a} - \boldsymbol{b}) - 2\big(2(\boldsymbol{a} - \boldsymbol{b})\big) = 2(\boldsymbol{a} + \boldsymbol{b})$

$\overrightarrow{MN} = 3\boldsymbol{a} - \boldsymbol{b} - 2(\boldsymbol{a} - \boldsymbol{b}) = \boldsymbol{a} + \boldsymbol{b} \qquad \overrightarrow{XY} = 2\overrightarrow{MN} \; they \; are \; therefore \; parallel$

37) Assign a number between 1 and 200 to every member of the population. Then create a list of random numbers between 1 and 200 using a calculator or spreadsheet. Finally match the numbers to names in the list.

38)

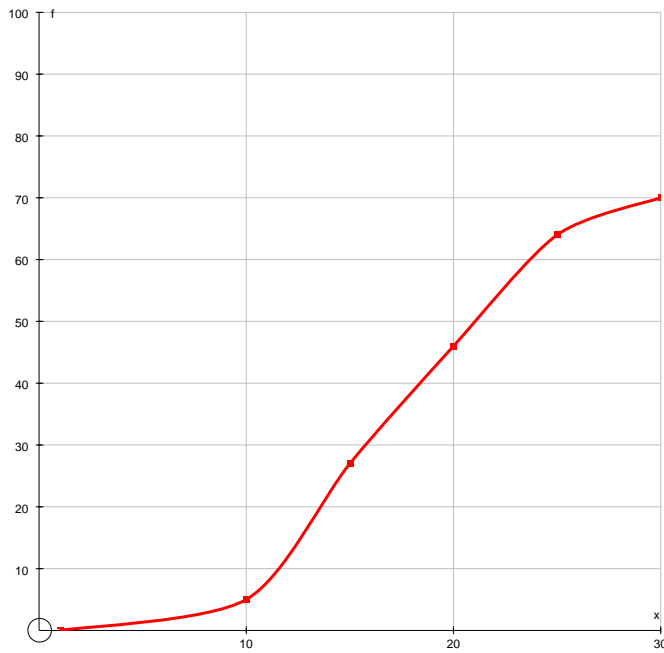| Weight 'w' in grams	Frequency
$0 < w \le 100$	50
$100 < w \le 150$	100
$150 < w \le 200$	150
$200 < w \le 250$	**90**
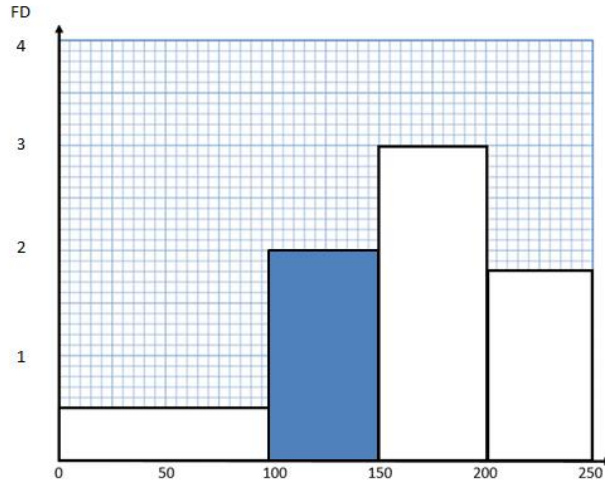
39) *Mean = 2   median = 2   mode = 1 and 2*

40) *Modal class: $10 < L \leq 15$*
    Estimate of mean: 17.18 cm (2dp)
    Cumulative frequency curve:

41) *Median = 25   LQ = 17   UQ = 34   IQR = 17*

42) $\dfrac{5}{7}$ $\dfrac{1}{14}$ $\dfrac{43}{70}$ $\dfrac{11}{35}$ $\dfrac{3}{7}$