A Level Further Mathematics Bridging Work

In order to achieve in A Level Further Mathematics it is **vital** that you have a secure knowledge of GCSE Mathematics content. In particular, you must be **fluent** in the following topics:

- Sketching quadratic graphs
- Solving simultaneous equations graphically
- Sketching cubic and reciprocal graphs
- Translating & Stretching graphs
- Straight line graphs
- Parallel and perpendicular lines
- Pythagoras' theorem
- Proportion
- Circle theorems
- Trigonometry in right-angled triangles
- The cosine rule & The sine rule
- Areas of triangles
- Rearranging equations
- Volume and surface area of 3D shapes
- Area under a graph

We expect that most students will already be confident in the vast majority of these topics. However, we are aware that students may have not finished the entire GCSE syllabus in Year 11 and may have significant gaps in understanding. It is essential that all FM students spend a significant amount of time practising these topics at regular intervals between the end of Year 11 and the start of Year 12.

Mathematical fluency does not simply mean that you have met this topic before and think that you remember how to do it. To reach fluency, you must be able to **quickly** and **accurately** recall concepts and methods.

Read the following instructions and complete all the work that you are instructed to complete below:

- This Bridging Work booklet is split into three sections A, B and C.
- **Section A** – this is a compilation of **'examples'** and explanations so firstly read through these examples and familiarise yourself with each topic and make any notes you feel appropriate.
- **Section B** – this section has all the work you need to complete. Once you feel confident then complete **all** the **'Practice'** and **'Extend' questions**. You must show your **full method**, a list of answers with no method will **not** be accepted.
- **Section C** – this section has the answers. Once you have completed Section B, **mark** your work from these answers to check you have understood each question.
- If you find that you have made mistakes, **identify** and **correct** these. Re-read the 'Examples' document to ensure that you have not misunderstood a concept.

Please bring all of your **completed and marked** bridging work to your first maths lesson where it will be checked by your maths teacher. We expect you to complete the questions on lined or squared paper, showing a **full method** and **working out**.

There will be an **assessment** covering these topics in the first week of Year 12. It is expected that all A Level Further Mathematics students will demonstrate an excellent understanding of all topics in this assessment.

Section A

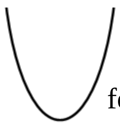
Examples

Sketching quadratic graphs

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- The graph of the quadratic function $y = ax^2 + bx + c$, where $a \neq 0$, is a curve called a parabola.
- Parabolas have a line of symmetry and a shape as shown.

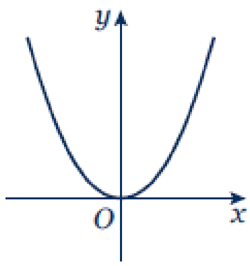

for $a > 0$


for $a < 0$

- To sketch the graph of a function, find the points where the graph intersects the axes.
- To find where the curve intersects the y-axis substitute $x = 0$ into the function.
- To find where the curve intersects the x-axis substitute $y = 0$ into the function.
- At the turning points of a graph the gradient of the curve is 0 and any tangents to the curve at these points are horizontal.
- To find the coordinates of the maximum or minimum point (turning points) of a quadratic curve (parabola) you can use the completed square form of the function.

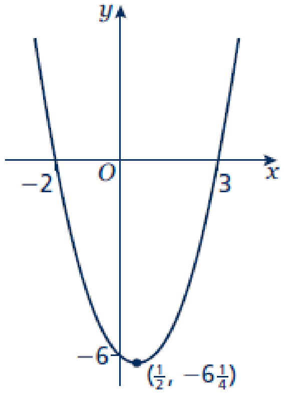
Examples

Example 1 Sketch the graph of $y = x^2$.

	The graph of $y = x^2$ is a parabola. When $x = 0$, $y = 0$. $a = 1$ which is greater than zero, so the graph has the shape: 

Example 2 Sketch the graph of $y = x^2 - x - 6$.

When $x = 0$, $y = 0^2 - 0 - 6 = -6$ So the graph intersects the y-axis at $(0, -6)$ When $y = 0$, $x^2 - x - 6 = 0$	**1** Find where the graph intersects the y-axis by substituting $x = 0$. **2** Find where the graph intersects the x-axis by substituting $y = 0$.
$(x + 2)(x - 3) = 0$	**3** Solve the equation by factorising.
$x = -2$ or $x = 3$	**4** Solve $(x + 2) = 0$ and $(x - 3) = 0$.
So, the graph intersects the x-axis at $(-2, 0)$ and $(3, 0)$	**5** $a = 1$ which is greater than zero, so the graph has the shape:  *(continued on next page)*

$x^2 - x - 6 = \left(x - \frac{1}{2}\right)^2 - \frac{1}{4} - 6$ $= \left(x - \frac{1}{2}\right)^2 - \frac{25}{4}$ When $\left(x - \frac{1}{2}\right)^2 = 0$, $x = \frac{1}{2}$ and $y = -\frac{25}{4}$, so the turning point is at the point $\left(\frac{1}{2}, -\frac{25}{4}\right)$  	**6** To find the turning point, complete the square. **7** The turning point is the minimum value for this expression and occurs when the term in the bracket is equal to zero.

Solving simultaneous equations graphically

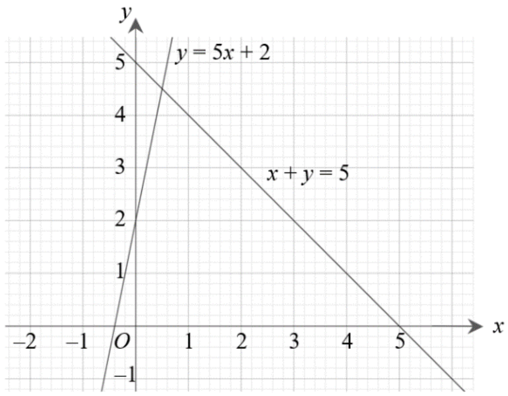
A LEVEL LINKS
Scheme of work: 1c. Equations – quadratic/linear simultaneous

Key points

- You can solve any pair of simultaneous equations by drawing the graph of both equations and finding the point/points of intersection.

Examples

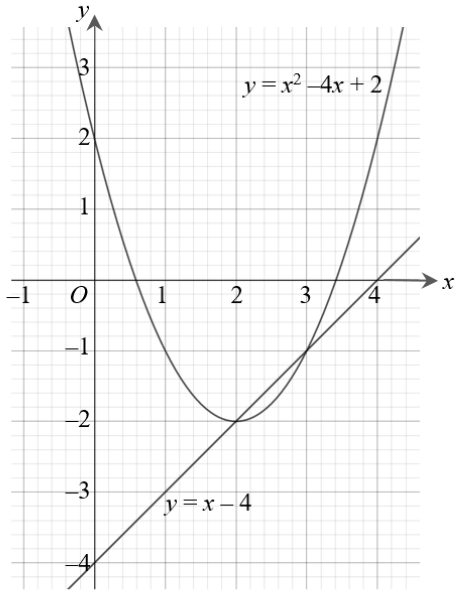
Example 1 Solve the simultaneous equations $y = 5x + 2$ and $x + y = 5$ graphically.

<table>
<tr><td>$y = 5 – x$</td><td>1 Rearrange the equation $x + y = 5$ to make y the subject.</td></tr>
<tr><td>$y = 5 – x$ has gradient –1 and y-intercept 5.
$y = 5x + 2$ has gradient 5 and y-intercept 2.
</td><td>2 Plot both graphs on the same grid using the gradients and y-intercepts.</td></tr>
<tr><td>Lines intersect at
$x = 0.5$, $y = 4.5$</td><td>3 The solutions of the simultaneous equations are the point of intersection.</td></tr>
<tr><td>Check:
First equation $y = 5x + 2$:
$4.5 = 5 \times 0.5 + 2$ YES
Second equation $x + y = 5$:
$0.5 + 4.5 = 5$ YES</td><td>4 Check your solutions by substituting the values into both equations.</td></tr>
</table>

Example 2 Solve the simultaneous equations $y = x - 4$ and $y = x^2 - 4x + 2$ graphically.

x	0	1	2	3	4
y	2	–1	–2	–1	2

1 Construct a table of values and calculate the points for the quadratic equation.



2 Plot the graph.

3 Plot the linear graph on the same grid using the gradient and *y*-intercept.
$y = x - 4$ has gradient 1 and *y*-intercept –4.

The line and curve intersect at
$x = 3$, $y = -1$ and $x = 2$, $y = -2$

4 The solutions of the simultaneous equations are the points of intersection.

Check:
First equation $y = x - 4$:
$-1 = 3 - 4$ YES
$-2 = 2 - 4$ YES
Second equation $y = x^2 - 4x + 2$:
$-1 = 3^2 - 4 \times 3 + 2$ YES
$-2 = 2^2 - 4 \times 2 + 2$ YES

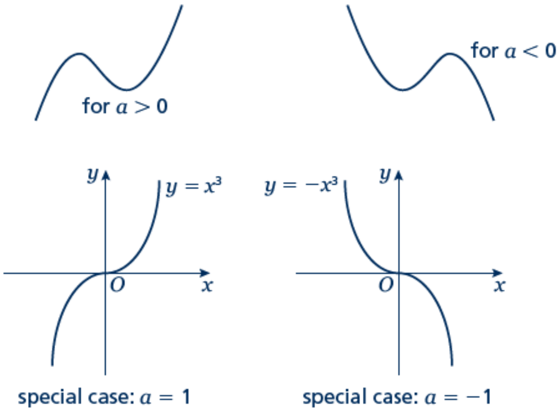
5 Check your solutions by substituting the values into both equations.

Sketching cubic and reciprocal graphs

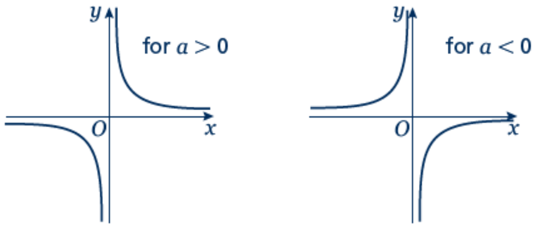
A LEVEL LINKS
Scheme of work: 1e. Graphs – cubic, quartic and reciprocal

Key points

- The graph of a cubic function, which can be written in the form $y = ax^3 + bx^2 + cx + d$, where $a \neq 0$, has one of the shapes shown here.



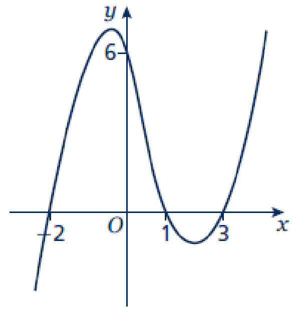
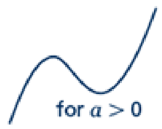
- The graph of a reciprocal function of the form $y = \frac{a}{x}$ has one of the shapes shown here.



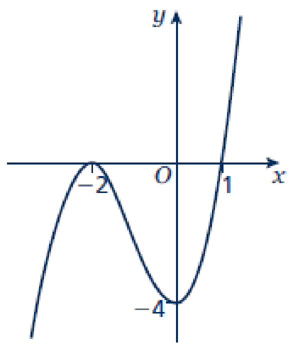
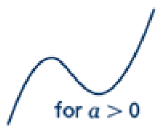
- To sketch the graph of a function, find the points where the graph intersects the axes.
- To find where the curve intersects the y-axis substitute $x = 0$ into the function.
- To find where the curve intersects the x-axis substitute $y = 0$ into the function.
- Where appropriate, mark and label the asymptotes on the graph.
- Asymptotes are lines (usually horizontal or vertical) which the curve gets closer to but never touches or crosses. Asymptotes usually occur with reciprocal functions. For example, the asymptotes for the graph of $y = \frac{a}{x}$ are the two axes (the lines $y = 0$ and $x = 0$).
- At the turning points of a graph the gradient of the curve is 0 and any tangents to the curve at these points are horizontal.
- A double root is when two of the solutions are equal. For example $(x - 3)^2(x + 2)$ has a double root at $x = 3$.
- When there is a double root, this is one of the turning points of a cubic function.

Examples

Example 1 Sketch the graph of $y = (x - 3)(x - 1)(x + 2)$

To sketch a cubic curve find intersects with both axes and use the key points above for the correct shape.	
When $x = 0$, $y = (0 - 3)(0 - 1)(0 + 2)$ $= (-3) \times (-1) \times 2 = 6$ The graph intersects the y-axis at (0, 6) When $y = 0$, $(x - 3)(x - 1)(x + 2) = 0$ So $x = 3$, $x = 1$ or $x = -2$ The graph intersects the x-axis at (−2, 0), (1, 0) and (3, 0) 	**1** Find where the graph intersects the axes by substituting $x = 0$ and $y = 0$. Make sure you get the coordinates the right way around, (x, y). **2** Solve the equation by solving $x - 3 = 0$, $x - 1 = 0$ and $x + 2 = 0$ **3** Sketch the graph. $a = 1 > 0$ so the graph has the shape: 

Example 2 Sketch the graph of $y = (x + 2)^2(x - 1)$

To sketch a cubic curve find intersects with both axes and use the key points above for the correct shape.	
When $x = 0$, $y = (0 + 2)^2(0 - 1)$ $= 2^2 \times (-1) = -4$ The graph intersects the y-axis at (0, −4) When $y = 0$, $(x + 2)^2(x - 1) = 0$ So $x = -2$ or $x = 1$ (−2, 0) is a turning point as $x = -2$ is a double root. The graph crosses the x-axis at (1, 0) 	**1** Find where the graph intersects the axes by substituting $x = 0$ and $y = 0$. **2** Solve the equation by solving $x + 2 = 0$ and $x - 1 = 0$ **3** $a = 1 > 0$ so the graph has the shape: 

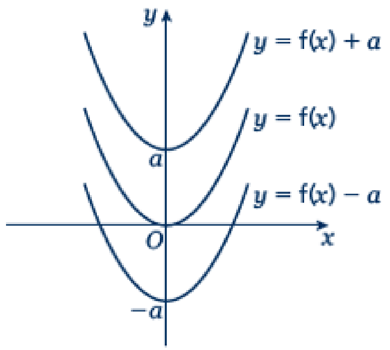
Translating graphs

A LEVEL LINKS
Scheme of work: 1f. Transformations – transforming graphs – f(x) notation

Key points

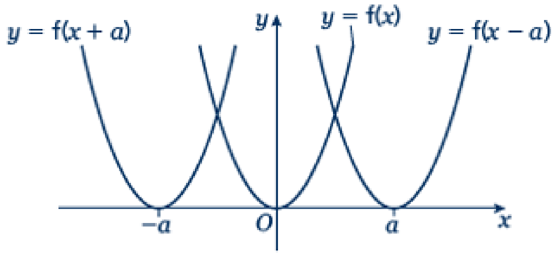
- The transformation $y = \text{f}(x) \pm a$ is a translation of $y = \text{f}(x)$ parallel to the y-axis; it is a vertical translation.

  As shown on the graph,
  - $y = \text{f}(x) + a$ translates $y = \text{f}(x)$ up
  - $y = \text{f}(x) - a$ translates $y = \text{f}(x)$ down.



- The transformation $y = \text{f}(x \pm a)$ is a translation of $y = \text{f}(x)$ parallel to the x-axis; it is a horizontal translation.

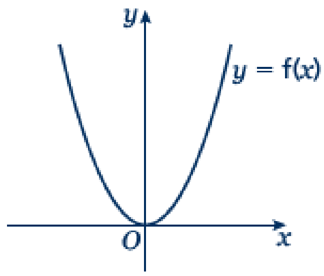
  As shown on the graph,
  - $y = \text{f}(x + a)$ translates $y = \text{f}(x)$ to the left
  - $y = \text{f}(x - a)$ translates $y = \text{f}(x)$ to the right.

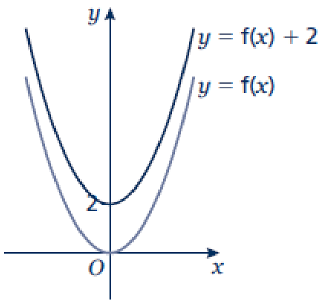


Examples

Example 1 The graph shows the function $y = \text{f}(x)$.

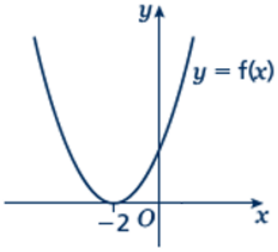
Sketch the graph of $y = \text{f}(x) + 2$.

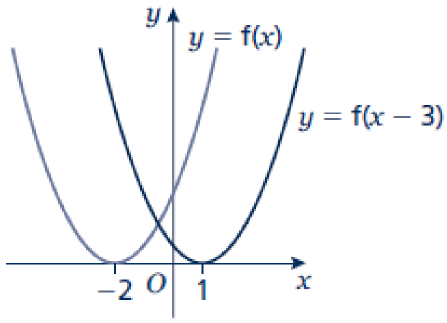


	For the function $y = \text{f}(x) + 2$ translate the function $y = \text{f}(x)$ 2 units up.

Example 2 The graph shows the function $y = f(x)$.

Sketch the graph of $y = f(x - 3)$.



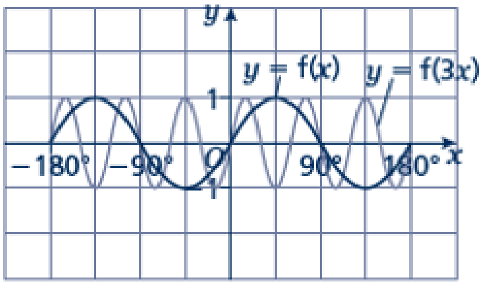
	For the function $y = f(x - 3)$ translate the function $y = f(x)$ 3 units right.

Stretching graphs

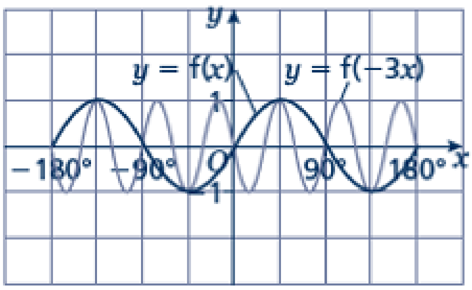
A LEVEL LINKS
Scheme of work: 1f. Transformations – transforming graphs – f(x) notation
Textbook: Pure Year 1, 4.6 Stretching graphs

Key points

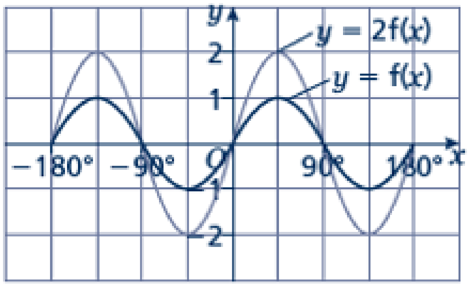
- The transformation $y = f(ax)$ is a horizontal stretch of $y = f(x)$ with scale factor $\frac{1}{a}$ parallel to the x-axis.



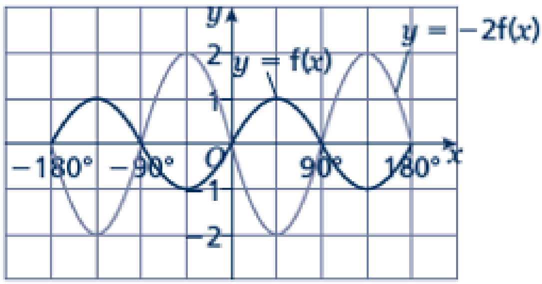
- The transformation $y = f(-ax)$ is a horizontal stretch of $y = f(x)$ with scale factor $\frac{1}{a}$ parallel to the x-axis and then a reflection in the y-axis.



- The transformation $y = af(x)$ is a vertical stretch of $y = f(x)$ with scale factor a parallel to the y-axis.



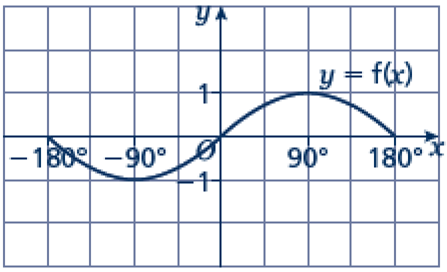
- The transformation $y = -af(x)$ is a vertical stretch of $y = f(x)$ with scale factor a parallel to the y-axis and then a reflection in the x-axis.

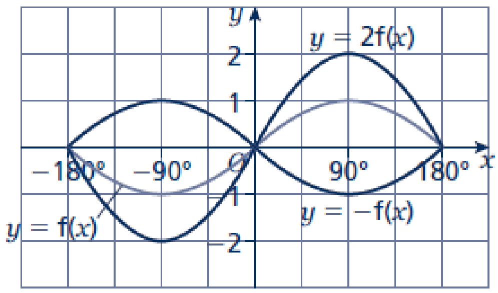


Examples

Example 3 The graph shows the function $y = \text{f}(x)$.

Sketch and label the graphs of
$y = 2\text{f}(x)$ and $y = -\text{f}(x)$.



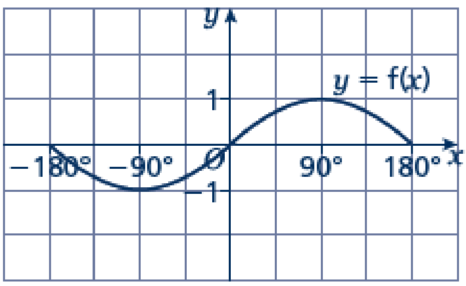


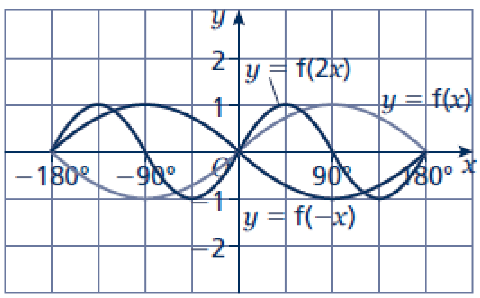
The function $y = 2\text{f}(x)$ is a vertical stretch of $y = \text{f}(x)$ with scale factor 2 parallel to the y-axis.

The function $y = -\text{f}(x)$ is a reflection of $y = \text{f}(x)$ in the x-axis.

Example 4 The graph shows the function $y = \text{f}(x)$.

Sketch and label the graphs of
$y = \text{f}(2x)$ and $y = \text{f}(-x)$.





The function $y = \text{f}(2x)$ is a horizontal stretch of $y = \text{f}(x)$ with scale factor $\frac{1}{2}$ parallel to the x-axis.

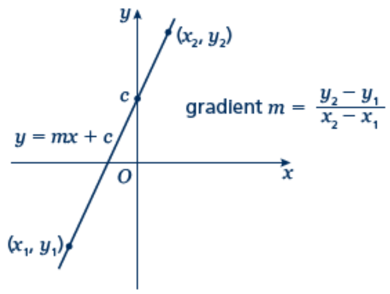
The function $y = \text{f}(-x)$ is a reflection of $y = \text{f}(x)$ in the y-axis.

Straight line graphs

A LEVEL LINKS
Scheme of work: 2a. Straight-line graphs, parallel/perpendicular, length and area problems

Key points

- A straight line has the equation $y = mx + c$, where m is the gradient and c is the y-intercept (where $x = 0$).
- The equation of a straight line can be written in the form $ax + by + c = 0$, where a, b and c are integers.
- When given the coordinates (x_1, y_1) and (x_2, y_2) of two points on a line the gradient is calculated using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$



Examples

Example 1 A straight line has gradient $-\frac{1}{2}$ and y-intercept 3.

Write the equation of the line in the form $ax + by + c = 0$.

$m = -\frac{1}{2}$ and $c = 3$ So $y = -\frac{1}{2}x + 3$ $\frac{1}{2}x + y - 3 = 0$ $x + 2y - 6 = 0$	**1** A straight line has equation $y = mx + c$. Substitute the gradient and y-intercept given in the question into this equation. **2** Rearrange the equation so all the terms are on one side and 0 is on the other side. **3** Multiply both sides by 2 to eliminate the denominator.

Example 2 Find the gradient and the y-intercept of the line with the equation $3y - 2x + 4 = 0$.

$3y - 2x + 4 = 0$ $3y = 2x - 4$ $y = \frac{2}{3}x - \frac{4}{3}$ Gradient $= m = \frac{2}{3}$ y-intercept $= c = -\frac{4}{3}$	**1** Make y the subject of the equation. **2** Divide all the terms by three to get the equation in the form $y = \ldots$ **3** In the form $y = mx + c$, the gradient is m and the y-intercept is c.

Example 3 Find the equation of the line which passes through the point (5, 13) and has gradient 3.

$m = 3$ $y = 3x + c$ $13 = 3 \times 5 + c$ $13 = 15 + c$ $c = -2$ $y = 3x - 2$	**1** Substitute the gradient given in the question into the equation of a straight line $y = mx + c$. **2** Substitute the coordinates $x = 5$ and $y = 13$ into the equation. **3** Simplify and solve the equation. **4** Substitute $c = -2$ into the equation $y = 3x + c$

Example 4 Find the equation of the line passing through the points with coordinates (2, 4) and (8, 7).

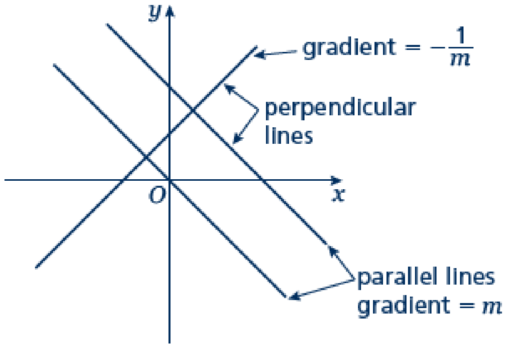
$x_1 = 2$, $x_2 = 8$, $y_1 = 4$ and $y_2 = 7$ $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-4}{8-2} = \frac{3}{6} = \frac{1}{2}$ $y = \frac{1}{2}x + c$ $4 = \frac{1}{2} \times 2 + c$ $c = 3$ $y = \frac{1}{2}x + 3$	**1** Substitute the coordinates into the equation $m = \frac{y_2 - y_1}{x_2 - x_1}$ to work out the gradient of the line. **2** Substitute the gradient into the equation of a straight line $y = mx + c$. **3** Substitute the coordinates of either point into the equation. **4** Simplify and solve the equation. **5** Substitute $c = 3$ into the equation $y = \frac{1}{2}x + c$

Parallel and perpendicular lines

A LEVEL LINKS
Scheme of work: 2a. Straight-line graphs, parallel/perpendicular, length and area problems

Key points

- When lines are parallel they have the same gradient.
- A line perpendicular to the line with equation $y = mx + c$ has gradient $-\frac{1}{m}$.



Examples

Example 1 Find the equation of the line parallel to $y = 2x + 4$ which passes through the point (4, 9).

$y = 2x + 4$ $m = 2$ $y = 2x + c$	**1** As the lines are parallel they have the same gradient. **2** Substitute $m = 2$ into the equation of a straight line $y = mx + c$.
$9 = 2 \times 4 + c$	**3** Substitute the coordinates into the equation $y = 2x + c$
$9 = 8 + c$ $c = 1$	**4** Simplify and solve the equation.
$y = 2x + 1$	**5** Substitute $c = 1$ into the equation $y = 2x + c$

Example 2 Find the equation of the line perpendicular to $y = 2x - 3$ which passes through the point (−2, 5).

$y = 2x - 3$ $m = 2$ $-\frac{1}{m} = -\frac{1}{2}$	**1** As the lines are perpendicular, the gradient of the perpendicular line is $-\frac{1}{m}$.
$y = -\frac{1}{2}x + c$	**2** Substitute $m = -\frac{1}{2}$ into $y = mx + c$.
$5 = -\frac{1}{2} \times (-2) + c$	**3** Substitute the coordinates (–2, 5) into the equation $y = -\frac{1}{2}x + c$
$5 = 1 + c$ $c = 4$	**4** Simplify and solve the equation.
$y = -\frac{1}{2}x + 4$	**5** Substitute $c = 4$ into $y = -\frac{1}{2}x + c$.

Example 3 A line passes through the points (0, 5) and (9, −1).
Find the equation of the line which is perpendicular to the line and passes through its midpoint.

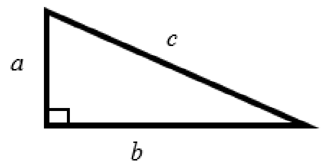
$x_1 = 0$, $x_2 = 9$, $y_1 = 5$ and $y_2 = -1$ $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-5}{9-0}$ $= \frac{-6}{9} = -\frac{2}{3}$	**1** Substitute the coordinates into the equation $m = \frac{y_2 - y_1}{x_2 - x_1}$ to work out the gradient of the line.
$-\frac{1}{m} = \frac{3}{2}$	**2** As the lines are perpendicular, the gradient of the perpendicular line is $-\frac{1}{m}$.
$y = \frac{3}{2}x + c$	**3** Substitute the gradient into the equation $y = mx + c$.
Midpoint = $\left(\frac{0+9}{2}, \frac{5+(-1)}{2}\right) = \left(\frac{9}{2}, 2\right)$	**4** Work out the coordinates of the midpoint of the line.
$2 = \frac{3}{2} \times \frac{9}{2} + c$	**5** Substitute the coordinates of the midpoint into the equation.
$c = -\frac{19}{4}$	**6** Simplify and solve the equation.
$y = \frac{3}{2}x - \frac{19}{4}$	**7** Substitute $c = -\frac{19}{4}$ into the equation $y = \frac{3}{2}x + c$.

Pythagoras' theorem

A LEVEL LINKS
Scheme of work: 2a. Straight-line graphs, parallel/perpendicular, length and area problems

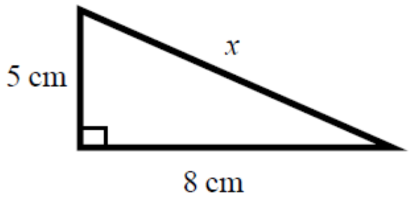
Key points

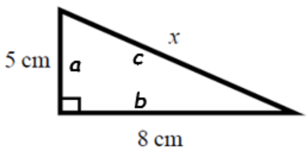
- In a right-angled triangle the longest side is called the hypotenuse.
- Pythagoras' theorem states that for a right-angled triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides.
  $c^2 = a^2 + b^2$



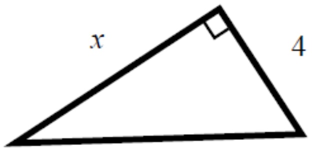
Examples

Example 1 Calculate the length of the hypotenuse.
Give your answer to 3 significant figures.



$c^2 = a^2 + b^2$ 	**1** Always start by stating the formula for Pythagoras' theorem and labelling the hypotenuse *c* and the other two sides *a* and *b*.
$x^2 = 5^2 + 8^2$ $x^2 = 25 + 64$ $x^2 = 89$ $x = \sqrt{89}$ $x = 9.433\ 981\ 13...$ $x = 9.43$ cm	**2** Substitute the values of *a*, *b* and *c* into the formula for Pythagoras' theorem. **3** Use a calculator to find the square root. **4** Round your answer to 3 significant figures and write the units with your answer.

Example 2 Calculate the length x.
Give your answer in surd form.



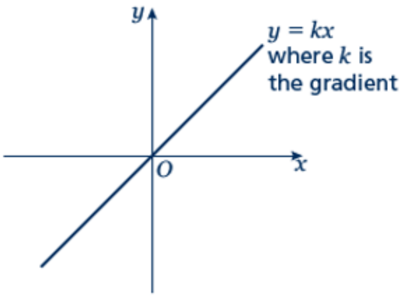
$c^2 = a^2 + b^2$ $10^2 = x^2 + 4^2$ $100 = x^2 + 16$ $x^2 = 84$ $x = \sqrt{84}$ $x = 2\sqrt{21}$ cm	**1** Always start by stating the formula for Pythagoras' theorem. **2** Substitute the values of *a*, *b* and *c* into the formula for Pythagoras' theorem. **3** Simplify the surd where possible and write the units in your answer.

Proportion

A LEVEL LINKS
Scheme of work: 2a. Straight-line graphs, parallel/perpendicular, length and area problems

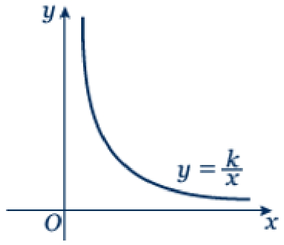
Key points

- Two quantities are in direct proportion when, as one quantity increases, the other increases at the same rate. Their ratio remains the same.
- '*y* is directly proportional to *x*' is written as $y \propto x$.
  If $y \propto x$ then $y = kx$, where k is a constant.
- When x is directly proportional to y, the graph is a straight line passing through the origin.



- Two quantities are in inverse proportion when, as one quantity increases, the other decreases at the same rate.
- '*y* is inversely proportional to *x*' is written as $y \propto \frac{1}{x}$.

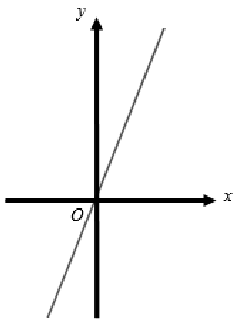
  If $y \propto \frac{1}{x}$ then $y = \frac{k}{x}$, where k is a constant.
- When x is inversely proportional to y the graph is the same shape as the graph of $y = \frac{1}{x}$



Examples

Example 1 *y* is directly proportional to *x*.
When $y = 16$, $x = 5$.
a Find x when $y = 30$.
b Sketch the graph of the formula.

a $y \propto x$	**1** Write y is directly proportional to x, using the symbol $\propto$.
$y = kx$	**2** Write the equation using k.
$16 = k \times 5$	**3** Substitute $y = 16$ and $x = 5$ into $y = kx$.
$k = 3.2$	**4** Solve the equation to find k.
$y = 3.2x$	**5** Substitute the value of k back into the equation $y = kx$.
When $y = 30$, $30 = 3.2 \times x$ $x = 9.375$	**6** Substitute $y = 30$ into $y = 3.2x$ and solve to find x when $y = 30$.

b 	**7** The graph of $y = 3.2x$ is a straight line passing through (0, 0) with a gradient of 3.2.

Example 2 y is directly proportional to x^2.
When $x = 3$, $y = 45$.
a Find y when $x = 5$.
b Find x when $y = 20$.

a $y \propto x^2$	**1** Write y is directly proportional to x^2, using the symbol $\propto$.
$y = kx^2$ $45 = k \times 3^2$	**2** Write the equation using k. **3** Substitute $y = 45$ and $x = 3$ into $y = kx^2$.
$k = 5$ $y = 5x^2$	**4** Solve the equation to find k. **5** Substitute the value of k back into the equation $y = kx^2$.
When $x = 5$, $y = 5 \times 5^2$ $y = 125$	**6** Substitute $x = 5$ into $y = 5x^2$ and solve to find y when $x = 5$.
b $20 = 5 \times x^2$ $x^2 = 4$ $x = \pm 2$	**7** Substitute $y = 20$ into $y = 5x^2$ and solve to find x when $y = 4$.

Example 3 P is inversely proportional to Q.
When $P = 100$, $Q = 10$.
Find Q when $P = 20$.

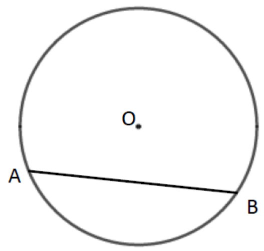
$P \propto \frac{1}{Q}$	**1** Write P is inversely proportional to Q, using the symbol $\propto$.
$P = \frac{k}{Q}$	**2** Write the equation using k.
$100 = \frac{k}{10}$	**3** Substitute $P = 100$ and $Q = 10$.
$k = 1000$	**4** Solve the equation to find k.
$P = \frac{1000}{Q}$	**5** Substitute the value of k into $P = \frac{k}{Q}$
$20 = \frac{1000}{Q}$ $Q = \frac{1000}{20} = 50$	**6** Substitute $P = 20$ into $P = \frac{1000}{Q}$ and solve to find Q when $P = 20$.

Circle theorems

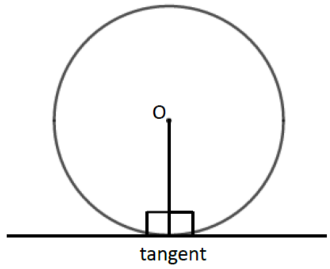
A LEVEL LINKS
Scheme of work: 2b. Circles – equation of a circle, geometric problems on a grid

Key points

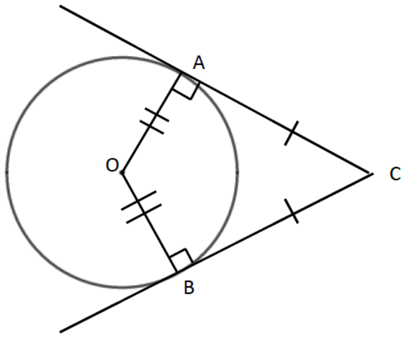
- A chord is a straight line joining two points on the circumference of a circle.
  So AB is a chord.



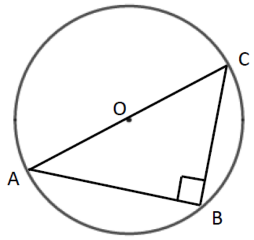
- A tangent is a straight line that touches the circumference of a circle at only one point.
  The angle between a tangent and the radius is 90°.



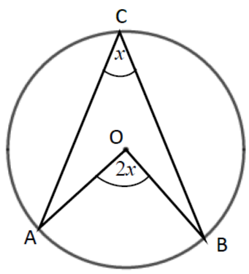
- Two tangents on a circle that meet at a point outside the circle are equal in length.
  So AC = BC.



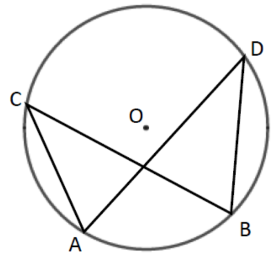
- The angle in a semicircle is a right angle.
  So angle ABC = 90°.



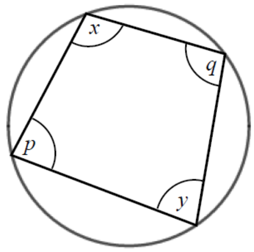
- When two angles are subtended by the same arc, the angle at the centre of a circle is twice the angle at the circumference.
  So angle AOB = 2 × angle ACB.



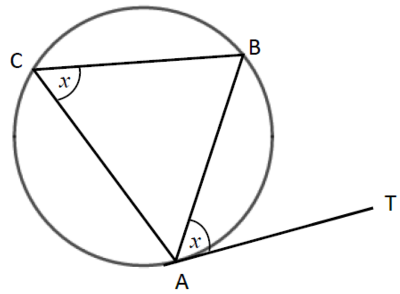
- Angles subtended by the same arc at the circumference are equal. This means that angles in the same segment are equal.
So angle ACB = angle ADB and
angle CAD = angle CBD.



- A cyclic quadrilateral is a quadrilateral with all four vertices on the circumference of a circle. Opposite angles in a cyclic quadrilateral total 180°.
So $x + y = 180°$ and $p + q = 180°$.

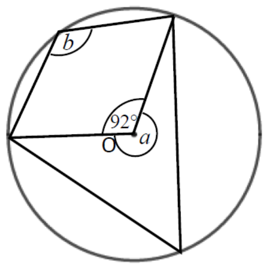


- The angle between a tangent and chord is equal to the angle in the alternate segment, this is known as the alternate segment theorem.
So angle BAT = angle ACB.



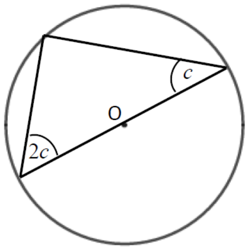
Examples

Example 1 Work out the size of each angle marked with a letter.
Give reasons for your answers.



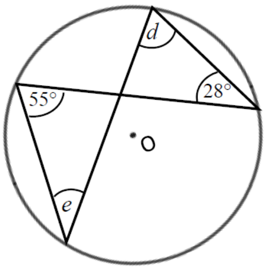
Angle $a = 360° - 92°$ $= 268°$ as the angles in a full turn total 360°.	**1** The angles in a full turn total 360°.
Angle $b = 268° \div 2$ $= 134°$ as when two angles are subtended by the same arc, the angle at the centre of a circle is twice the angle at the circumference.	**2** Angles *a* and *b* are subtended by the same arc, so angle *b* is half of angle *a*.

Example 2 Work out the size of the angles in the triangle.
Give reasons for your answers.



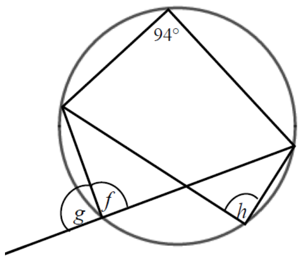
Angles are 90°, $2c$ and c. $90° + 2c + c = 180°$ $90° + 3c = 180°$ $3c = 90°$ $c = 30°$ $2c = 60°$ The angles are 30°, 60° and 90° as the angle in a semi-circle is a right angle and the angles in a triangle total 180°.	**1** The angle in a semicircle is a right angle. **2** Angles in a triangle total 180°. **3** Simplify and solve the equation.

Example 3 Work out the size of each angle marked with a letter.
Give reasons for your answers.



Angle d = 55° as angles subtended by the same arc are equal. Angle e = 28° as angles subtended by the same arc are equal.	**1** Angles subtended by the same arc are equal so angle 55° and angle d are equal. **2** Angles subtended by the same arc are equal so angle 28° and angle e are equal.

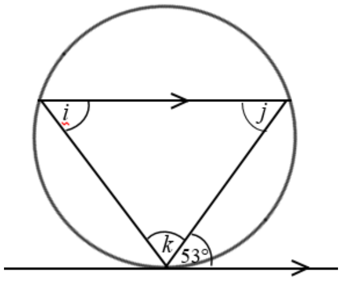
Example 4 Work out the size of each angle marked with a letter.
Give reasons for your answers.



Angle $f = 180° - 94°$ $= 86°$ as opposite angles in a cyclic quadrilateral total 180°.	**1** Opposite angles in a cyclic quadrilateral total 180° so angle 94° and angle f total 180°. *(continued on next page)*

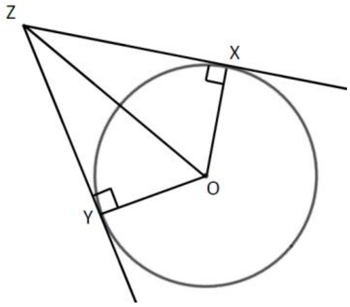
Angle $g = 180° - 86°$ $= 84°$ as angles on a straight line total 180°. Angle h = angle f = 86° as angles subtended by the same arc are equal.	**2** Angles on a straight line total 180° so angle f and angle g total 180°. **3** Angles subtended by the same arc are equal so angle f and angle h are equal.

Example 5 Work out the size of each angle marked with a letter. Give reasons for your answers.



Angle i = 53° because of the alternate segment theorem. Angle j = 53° because it is the alternate angle to 53°. Angle $k = 180° - 53° - 53°$ $= 74°$ as angles in a triangle total 180°.	**1** The angle between a tangent and chord is equal to the angle in the alternate segment. **2** As there are two parallel lines, angle 53° is equal to angle j because they are alternate angles. **3** The angles in a triangle total 180°, so $i + j + k = 180°$.

Example 6 XZ and YZ are two tangents to a circle with centre O. Prove that triangles XZO and YZO are congruent.



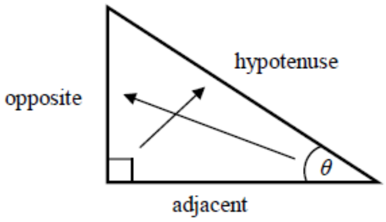
Angle OXZ = 90° and angle OYZ = 90° as the angles in a semicircle are right angles. OZ is a common line and is the hypotenuse in both triangles. OX = OY as they are radii of the same circle. So triangles XZO and YZO are congruent, RHS.	For two triangles to be congruent you need to show one of the following. • All three corresponding sides are equal (SSS). • Two corresponding sides and the included angle are equal (SAS). • One side and two corresponding angles are equal (ASA). • A right angle, hypotenuse and a shorter side are equal (RHS).

Trigonometry in right-angled triangles

A LEVEL LINKS
Scheme of work: 4a. Trigonometric ratios and graphs

Key points

- In a right-angled triangle:
  - the side opposite the right angle is called the hypotenuse
  - the side opposite the angle θ is called the opposite
  - the side next to the angle θ is called the adjacent.



- In a right-angled triangle:
  - the ratio of the opposite side to the hypotenuse is the sine of angle θ, $\sin\theta = \frac{\text{opp}}{\text{hyp}}$
  - the ratio of the adjacent side to the hypotenuse is the cosine of angle θ, $\cos\theta = \frac{\text{adj}}{\text{hyp}}$
  - the ratio of the opposite side to the adjacent side is the tangent of angle θ, $\tan\theta = \frac{\text{opp}}{\text{adj}}$

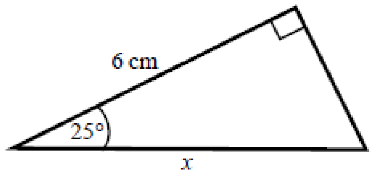
- If the lengths of two sides of a right-angled triangle are given, you can find a missing angle using the inverse trigonometric functions: $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$.

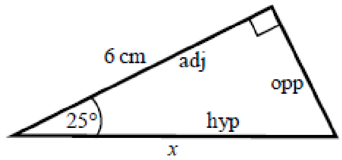
- The sine, cosine and tangent of some angles may be written exactly.

	0	**30°**	**45°**	**60°**	**90°**
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	

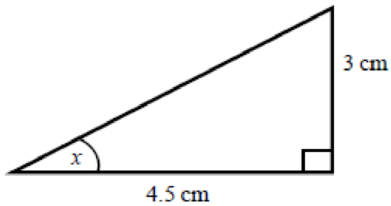
Examples

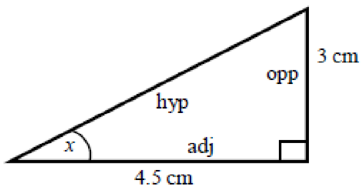
Example 1 Calculate the length of side x.
Give your answer correct to 3 significant figures.



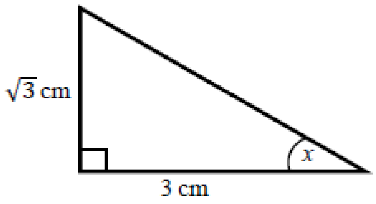
	**1** Always start by labelling the sides.
$\cos\theta = \frac{\text{adj}}{\text{hyp}}$	**2** You are given the adjacent and the hypotenuse so use the cosine ratio.
$\cos 25° = \frac{6}{x}$	**3** Substitute the sides and angle into the cosine ratio.
$x = \frac{6}{\cos 25°}$	**4** Rearrange to make x the subject.
x = 6.620 267 5...	**5** Use your calculator to work out 6 ÷ cos 25°.
x = 6.62 cm	**6** Round your answer to 3 significant figures and write the units in your answer.

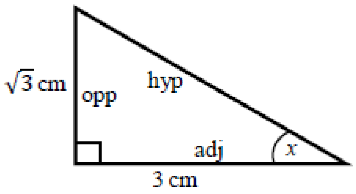
Example 2 Calculate the size of angle x.
Give your answer correct to 3 significant figures.



	**1** Always start by labelling the sides.
$\tan\theta = \frac{\text{opp}}{\text{adj}}$	**2** You are given the opposite and the adjacent so use the tangent ratio.
$\tan x = \frac{3}{4.5}$	**3** Substitute the sides and angle into the tangent ratio.
$x = \tan^{-1}\left(\frac{3}{4.5}\right)$	**4** Use $\tan^{-1}$ to find the angle.
x = 33.690 067 5...	**5** Use your calculator to work out $\tan^{-1}(3 \div 4.5)$.
x = 33.7°	**6** Round your answer to 3 significant figures and write the units in your answer.

Example 3 Calculate the exact size of angle x.



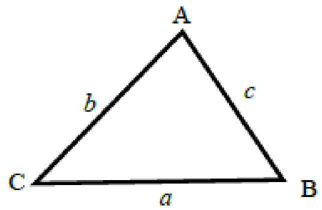
	**1** Always start by labelling the sides.
$\tan\theta = \frac{\text{opp}}{\text{adj}}$	**2** You are given the opposite and the adjacent so use the tangent ratio.
$\tan x = \frac{\sqrt{3}}{3}$	**3** Substitute the sides and angle into the tangent ratio.
$x = 30°$	**4** Use the table from the key points to find the angle.

The cosine rule

A LEVEL LINKS
Scheme of work: 4a. Trigonometric ratios and graphs
Textbook: Pure Year 1, 9.1 The cosine rule

Key points

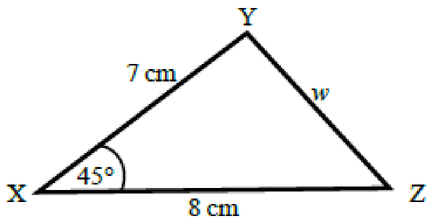
- *a* is the side opposite angle A.
  b is the side opposite angle B.
  c is the side opposite angle C.

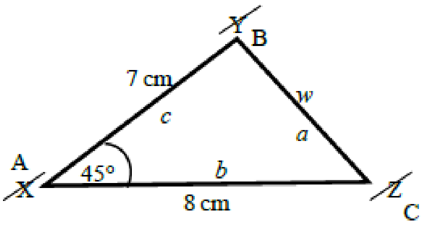


- You can use the cosine rule to find the length of a side when two sides and the included angle are given.
- To calculate an unknown side use the formula $a^2 = b^2 + c^2 - 2bc\cos A$.
- Alternatively, you can use the cosine rule to find an unknown angle if the lengths of all three sides are given.
- To calculate an unknown angle use the formula $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$.

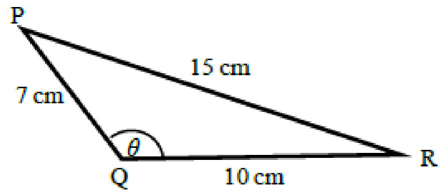
Examples

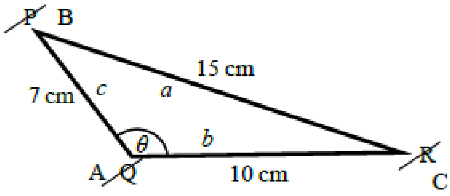
Example 4 Work out the length of side *w*.
Give your answer correct to 3 significant figures.



	**1** Always start by labelling the angles and sides.
$a^2 = b^2 + c^2 - 2bc\cos A$	**2** Write the cosine rule to find the side.
$w^2 = 8^2 + 7^2 - 2 \times 8 \times 7 \times \cos 45°$	**3** Substitute the values *a*, *b* and *A* into the formula.
w^2 = 33.804 040 51... $w = \sqrt{33.80404051}$	**4** Use a calculator to find w^2 and then *w*.
w = 5.81 cm	**5** Round your final answer to 3 significant figures and write the units in your answer.

Example 5 Work out the size of angle θ.
Give your answer correct to 1 decimal place.



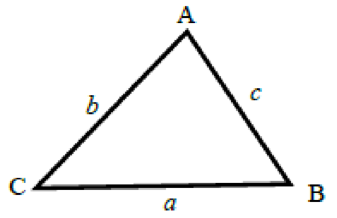
	**1** Always start by labelling the angles and sides.
$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$	**2** Write the cosine rule to find the angle.
$\cos\theta = \dfrac{10^2 + 7^2 - 15^2}{2 \times 10 \times 7}$	**3** Substitute the values *a*, *b* and *c* into the formula.
$\cos\theta = \dfrac{-76}{140}$	**4** Use $\cos^{-1}$ to find the angle.
θ = 122.878 349...	**5** Use your calculator to work out $\cos^{-1}(-76 \div 140)$.
θ = 122.9°	**6** Round your answer to 1 decimal place and write the units in your answer.

The sine rule

A LEVEL LINKS
Scheme of work: 4a. Trigonometric ratios and graphs
Textbook: Pure Year 1, 9.2 The sine rule

Key points

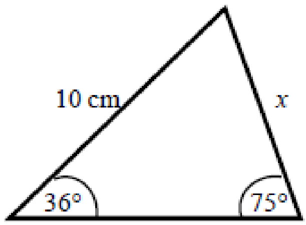
- *a* is the side opposite angle A.
  b is the side opposite angle B.
  c is the side opposite angle C.

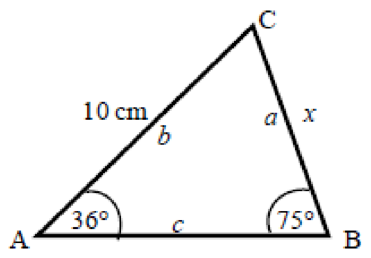


- You can use the sine rule to find the length of a side when its opposite angle and another opposite side and angle are given.
- To calculate an unknown side use the formula $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.
- Alternatively, you can use the sine rule to find an unknown angle if the opposite side and another opposite side and angle are given.
- To calculate an unknown angle use the formula $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.

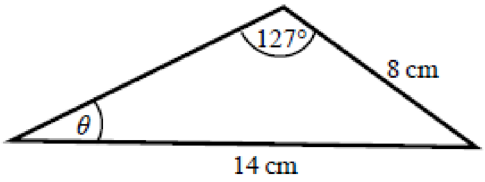
Examples

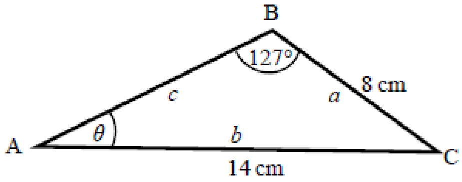
Example 6 Work out the length of side *x*.
Give your answer correct to 3 significant figures.



	**1** Always start by labelling the angles and sides.
$\frac{a}{\sin A} = \frac{b}{\sin B}$	**2** Write the sine rule to find the side.
$\frac{x}{\sin 36°} = \frac{10}{\sin 75°}$	**3** Substitute the values *a*, *b*, *A* and *B* into the formula.
$x = \frac{10 \times \sin 36°}{\sin 75°}$	**4** Rearrange to make *x* the subject.
$x = 6.09$ cm	**5** Round your answer to 3 significant figures and write the units in your answer.

Example 7 Work out the size of angle θ.
Give your answer correct to 1 decimal place.



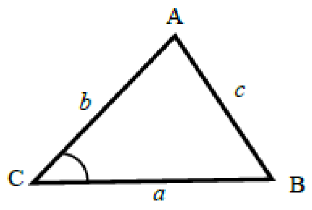
	**1** Always start by labelling the angles and sides.
$\frac{\sin A}{a} = \frac{\sin B}{b}$	**2** Write the sine rule to find the angle.
$\frac{\sin\theta}{8} = \frac{\sin 127°}{14}$	**3** Substitute the values *a*, *b*, *A* and *B* into the formula.
$\sin\theta = \frac{8 \times \sin 127°}{14}$	**4** Rearrange to make sin θ the subject.
$\theta = 27.2°$	**5** Use $\sin^{-1}$ to find the angle. Round your answer to 1 decimal place and write the units in your answer.

Areas of triangles

A LEVEL LINKS
Scheme of work: 4a. Trigonometric ratios and graphs
Textbook: Pure Year 1, 9.3 Areas of triangles

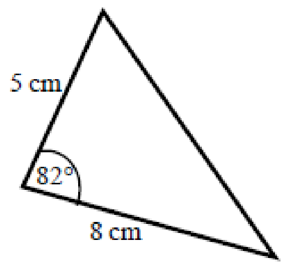
Key points

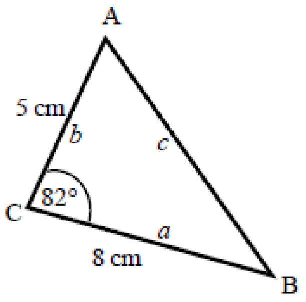
- *a* is the side opposite angle A.
  b is the side opposite angle B.
  c is the side opposite angle C.
- The area of the triangle is $\frac{1}{2}ab\sin C$.



Examples

Example 8 Find the area of the triangle.



	**1** Always start by labelling the sides and angles of the triangle.
Area = $\frac{1}{2}ab\sin C$	**2** State the formula for the area of a triangle.
Area = $\frac{1}{2}\times 8\times 5\times \sin 82°$	**3** Substitute the values of *a*, *b* and *C* into the formula for the area of a triangle.
Area = 19.805 361...	**4** Use a calculator to find the area.
Area = 19.8 cm^2	**5** Round your answer to 3 significant figures and write the units in your answer.

Rearranging equations

A LEVEL LINKS
Scheme of work: 6a. Definition, differentiating polynomials, second derivatives
Textbook: Pure Year 1, 12.1 Gradients of curves

Key points

- To change the subject of a formula, get the terms containing the subject on one side and everything else on the other side.
- You may need to factorise the terms containing the new subject.

Examples

Example 1 Make *t* the subject of the formula $v = u + at$.

$v = u + at$ $v - u = at$ $t = \frac{v-u}{a}$	**1** Get the terms containing *t* on one side and everything else on the other side. **2** Divide throughout by *a*.

Example 2 Make *t* the subject of the formula $r = 2t - \pi t$.

$r = 2t - \pi t$ $r = t(2 - \pi)$ $t = \frac{r}{2-\pi}$	**1** All the terms containing *t* are already on one side and everything else is on the other side. **2** Factorise as *t* is a common factor. **3** Divide throughout by $2 - \pi$.

Example 3 Make *t* the subject of the formula $\frac{t+r}{5} = \frac{3t}{2}$.

$\frac{t+r}{5} = \frac{3t}{2}$ $2t + 2r = 15t$ $2r = 13t$ $t = \frac{2r}{13}$	**1** Remove the fractions first by multiplying throughout by 10. **2** Get the terms containing *t* on one side and everything else on the other side and simplify. **3** Divide throughout by 13.

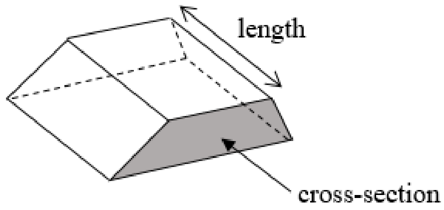
Example 4 Make t the subject of the formula $r = \frac{3t+5}{t-1}$.

$r = \frac{3t+5}{t-1}$	**1** Remove the fraction first by multiplying throughout by $t - 1$.
$r(t - 1) = 3t + 5$	**2** Expand the brackets.
$rt - r = 3t + 5$	**3** Get the terms containing t on one side and everything else on the other side.
$rt - 3t = 5 + r$	
$t(r - 3) = 5 + r$	**4** Factorise the LHS as t is a common factor.
$t = \frac{5+r}{r-3}$	**5** Divide throughout by $r - 3$.

Volume and surface area of 3D shapes

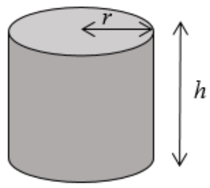
A LEVEL LINKS
Scheme of work: 6b. Gradients, tangents, normals, maxima and minima

Key points

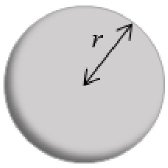


- Volume of a prism = cross-sectional area × length.
- The surface area of a 3D shape is the total area of all its faces.

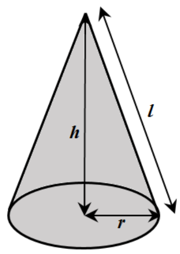
- Volume of a pyramid = $\frac{1}{3}$ × area of base × vertical height.



- Volume of a cylinder = $\pi r^2 h$
- Total surface area of a cylinder = $2\pi r^2 + 2\pi rh$



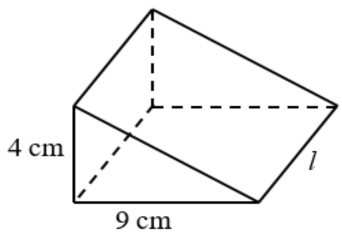
- Volume of a sphere = $\frac{4}{3}\pi r^3$
- Surface area of a sphere = $4\pi r^2$



- Volume of a cone = $\frac{1}{3}\pi r^2 h$
- Total surface area of a cone = $\pi rl + \pi r^2$

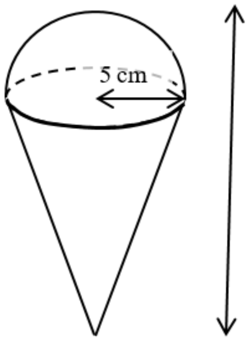
Examples

Example 1 The triangular prism has volume 504 cm^3. Work out its length.



$V = \frac{1}{2}bhl$	**1** Write out the formula for the volume of a triangular prism.
$504 = \frac{1}{2} \times 9 \times 4 \times l$	**2** Substitute known values into the formula.
$504 = 18 \times l$	**3** Simplify
$l = 504 \div 18$	**4** Rearrange to work out l.
= 28 cm	**5** Remember the units.

Example 2 Calculate the volume of the 3D solid.
Give your answer in terms of π.



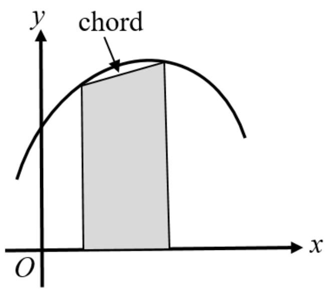
Total volume = volume of hemisphere + Volume of cone $= \frac{1}{2}$ of $\frac{4}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$	**1** The solid is made up of a hemisphere radius 5 cm and a cone with radius 5 cm and height 12 − 5 = 7 cm.
Total volume = $\frac{1}{2} \times \frac{4}{3} \times \pi \times 5^3 + \frac{1}{3} \times \pi \times 5^2 \times 7$	**2** Substitute the measurements into the formula for the total volume.
$= \frac{425}{3}\pi$ cm^3	**3** Remember the units.

Area under a graph

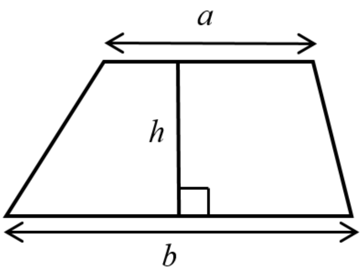
A LEVEL LINKS
Scheme of work: 7b. Definite integrals and areas under curves

Key points

- To estimate the area under a curve, draw a chord between the two points you are finding the area between and straight lines down to the horizontal axis to create a trapezium. The area of the trapezium is an approximation for the area under a curve.

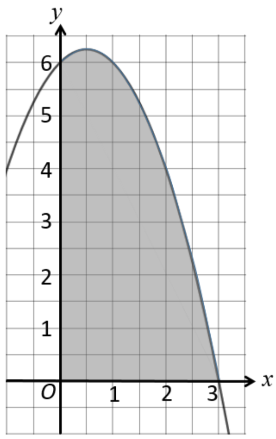


- The area of a trapezium = $\frac{1}{2}h(a+b)$



Examples

Example 1 Estimate the area of the region between the curve $y = (3 - x)(2 + x)$ and the x-axis from $x = 0$ to $x = 3$. Use three strips of width 1 unit.



x	0	1	2	3
***y* = (3 − *x*)(2 + *x*)**	6	6	4	0

Trapezium 1:
$a_1 = 6 - 0 = 6$, $b_1 = 6 - 0 = 6$
Trapezium 2:
$a_2 = 6 - 0 = 6$, $b_2 = 4 - 0 = 4$
Trapezium 3:
$a_3 = 4 - 0 = 4$, $a_3 = 0 - 0 = 0$

1 Use a table to record the value of y on the curve for each value of x.

2 Work out the dimensions of each trapezium. The distances between the y-values on the curve and the x-axis give the values for a.

(continued on next page)

$\frac{1}{2}h(a_1 + b_1) = \frac{1}{2} \times 1(6+6) = 6$

$\frac{1}{2}h(a_2 + b_2) = \frac{1}{2} \times 1(6+4) = 5$

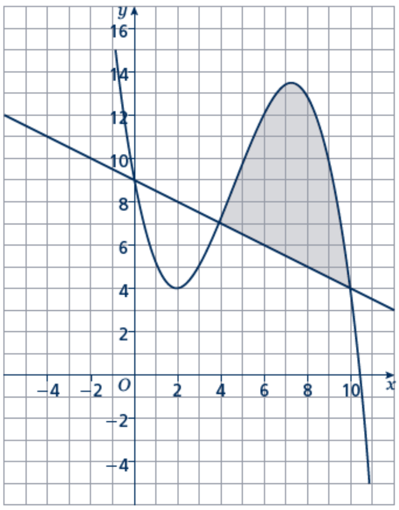
$\frac{1}{2}h(a_3 + b_3) = \frac{1}{2} \times 1(4+0) = 2$

3 Work out the area of each trapezium. h = 1 since the width of each trapezium is 1 unit.

Area = 6 + 5 + 2 = 13 units^2

4 Work out the total area. Remember to give units with your answer.

Example 2 Estimate the shaded area.
Use three strips of width 2 units.



x	4	6	8	10
y	7	12	13	4

1 Use a table to record y on the curve for each value of x.

x	4	6	8	10
y	7	6	5	4

2 Use a table to record y on the straight line for each value of x.

Trapezium 1:
$a_1 = 7 - 7 = 0$, $b_1 = 12 - 6 = 6$
Trapezium 2:
$a_2 = 12 - 6 = 6$, $b_2 = 13 - 5 = 8$
Trapezium 3:
$a_3 = 13 - 5 = 8$, $a_3 = 4 - 4 = 0$

3 Work out the dimensions of each trapezium. The distances between the y-values on the curve and the y-values on the straight line give the values for a.

$\frac{1}{2}h(a_1 + b_1) = \frac{1}{2} \times 2(0+6) = 6$

$\frac{1}{2}h(a_2 + b_2) = \frac{1}{2} \times 2(6+8) = 14$

$\frac{1}{2}h(a_3 + b_3) = \frac{1}{2} \times 2(8+0) = 8$

4 Work out the area of each trapezium. h = 2 since the width of each trapezium is 2 units.

Area = 6 + 14 + 8 = 28 units^2

5 Work out the total area. Remember to give units with your answer.

Section B

Practice and Extend Questions

Sketching quadratic graphs

Practice

1 Sketch the graph of $y = -x^2$.

2 Sketch each graph, labelling where the curve crosses the axes.

a $y = (x + 2)(x - 1)$ **b** $y = x(x - 3)$ **c** $y = (x + 1)(x + 5)$

3 Sketch each graph, labelling where the curve crosses the axes.

a $y = x^2 - x - 6$ **b** $y = x^2 - 5x + 4$ **c** $y = x^2 - 4$

d $y = x^2 + 4x$ **e** $y = 9 - x^2$ **f** $y = x^2 + 2x - 3$

4 Sketch the graph of $y = 2x^2 + 5x - 3$, labelling where the curve crosses the axes.

Extend

5 Sketch each graph. Label where the curve crosses the axes and write down the coordinates of the turning point.

a $y = x^2 - 5x + 6$ **b** $y = -x^2 + 7x - 12$ **c** $y = -x^2 + 4x$

6 Sketch the graph of $y = x^2 + 2x + 1$. Label where the curve crosses the axes and write down the equation of the line of symmetry.

Solving simultaneous equations graphically

Practice

1 Solve these pairs of simultaneous equations graphically.

a $y = 3x - 1$ and $y = x + 3$

b $y = x - 5$ and $y = 7 - 5x$

c $y = 3x + 4$ and $y = 2 - x$

2 Solve these pairs of simultaneous equations graphically.

a $x + y = 0$ and $y = 2x + 6$

b $4x + 2y = 3$ and $y = 3x - 1$

c $2x + y + 4 = 0$ and $2y = 3x - 1$

Hint

Rearrange the equation to make y the subject.

3 Solve these pairs of simultaneous equations graphically.

a $y = x - 1$ and $y = x^2 - 4x + 3$

b $y = 1 - 3x$ and $y = x^2 - 3x - 3$

c $y = 3 - x$ and $y = x^2 + 2x + 5$

4 Solve the simultaneous equations $x + y = 1$ and $x^2 + y^2 = 25$ graphically.

Extend

5 **a** Solve the simultaneous equations $2x + y = 3$ and $x^2 + y = 4$

i graphically

ii algebraically to 2 decimal places.

b Which method gives the more accurate solutions? Explain your answer.

Sketching cubic and reciprocal graphs

Practice

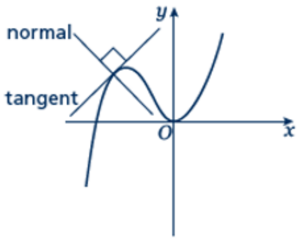
1 Here are six equations.

A	$y = \frac{5}{x}$	**B**	$y = x^2 + 3x - 10$	**C**	$y = x^3 + 3x^2$
D	$y = 1 - 3x^2 - x^3$	**E**	$y = x^3 - 3x^2 - 1$	**F**	$x + y = 5$

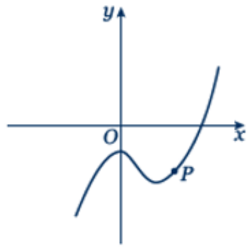
> **Hint**
>
> Find where each of the cubic equations cross the y-axis.

Here are six graphs.

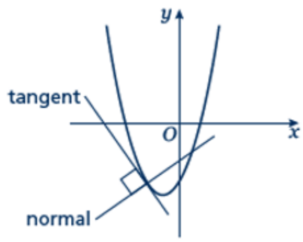
i



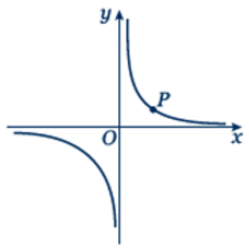
ii



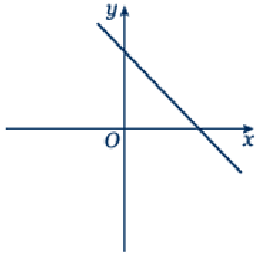
iii



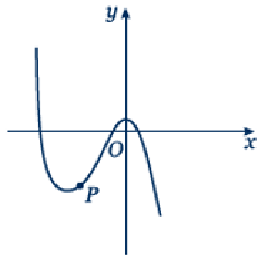
iv



v



vi



a Match each graph to its equation.

b Copy the graphs ii, iv and vi and draw the tangent and normal each at point P.

Sketch the following graphs

2 $y = 2x^3$

3 $y = x(x - 2)(x + 2)$

4 $y = (x + 1)(x + 4)(x - 3)$

5 $y = (x + 1)(x - 2)(1 - x)$

6 $y = (x - 3)^2(x + 1)$

7 $y = (x - 1)^2(x - 2)$

8 $y = \frac{3}{x}$

> **Hint:** Look at the shape of $y = \frac{a}{x}$ in the second key point.

9 $y = -\frac{2}{x}$

Extend

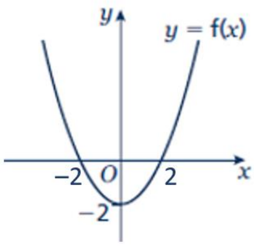
10 Sketch the graph of $y = \frac{1}{x+2}$

11 Sketch the graph of $y = \frac{1}{x-1}$

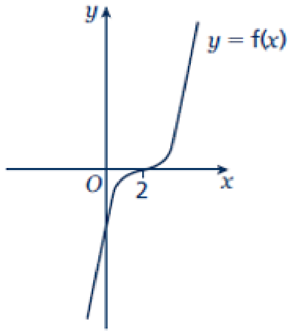
Translating graphs

Practice

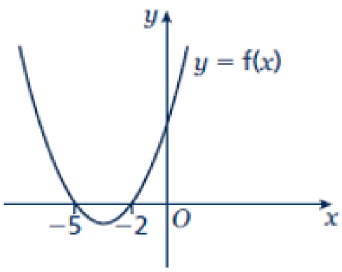
1 The graph shows the function $y = f(x)$.
Copy the graph and on the same axes sketch and label the graphs of $y = f(x) + 4$ and $y = f(x + 2)$.



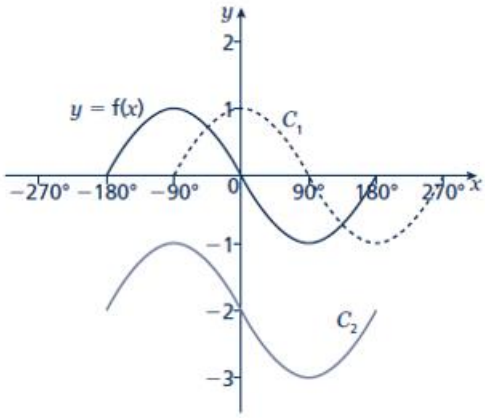
2 The graph shows the function $y = f(x)$.
Copy the graph and on the same axes sketch and label the graphs of $y = f(x + 3)$ and $y = f(x) – 3$.



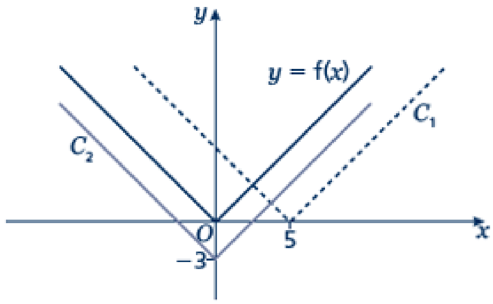
3 The graph shows the function $y = f(x)$.
Copy the graph and on the same axes sketch the graph of $y = f(x – 5)$.



4 The graph shows the function $y = \mathrm{f}(x)$ and two transformations of $y = \mathrm{f}(x)$, labelled C_1 and C_2. Write down the equations of the translated curves C_1 and C_2 in function form.



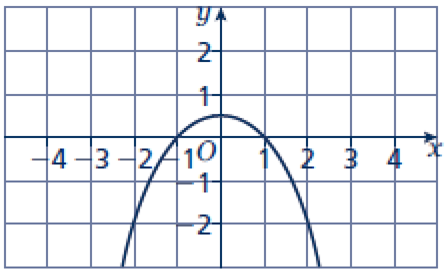
5 The graph shows the function $y = \mathrm{f}(x)$ and two transformations of $y = \mathrm{f}(x)$, labelled C_1 and C_2. Write down the equations of the translated curves C_1 and C_2 in function form.



6 The graph shows the function $y = \mathrm{f}(x)$.

a Sketch the graph of $y = \mathrm{f}(x) + 2$

b Sketch the graph of $y = \mathrm{f}(x + 2)$



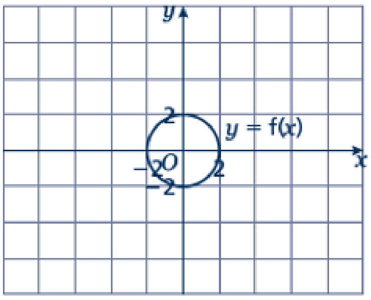
Stretching graphs

Practice

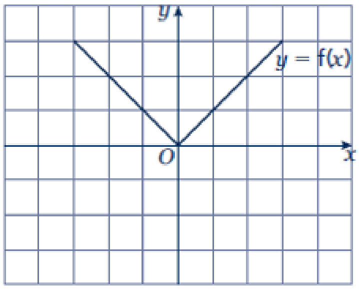
7 The graph shows the function $y = f(x)$.

a Copy the graph and on the same axes sketch and label the graph of $y = 3f(x)$.

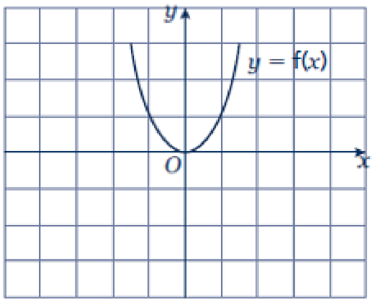
b Make another copy of the graph and on the same axes sketch and label the graph of $y = f(2x)$.



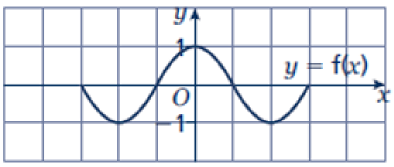
8 The graph shows the function $y = f(x)$.
Copy the graph and on the same axes sketch and label the graphs of
$y = –2f(x)$ and $y = f(3x)$.



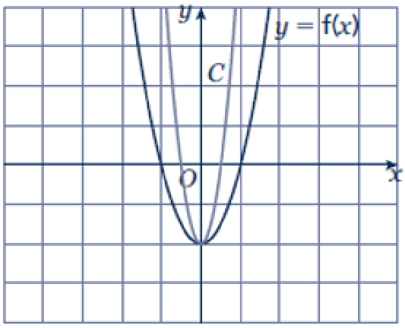
9 The graph shows the function $y = f(x)$.
Copy the graph and, on the same axes, sketch and label the graphs of
$y = –f(x)$ and $y = f\left(\frac{1}{2}x\right)$.



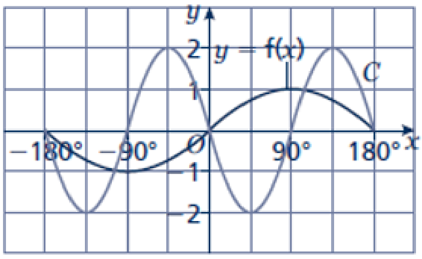
10 The graph shows the function $y = f(x)$.
Copy the graph and, on the same axes, sketch the graph of $y = –f(2x)$.



11 The graph shows the function $y = f(x)$ and a transformation, labelled C.
Write down the equation of the translated curve C in function form.



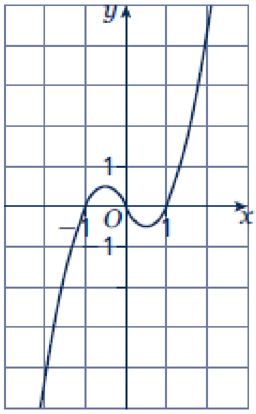
12 The graph shows the function $y = \mathrm{f}(x)$ and a transformation labelled C.
Write down the equation of the translated curve C in function form.



13 The graph shows the function $y = \mathrm{f}(x)$.

a Sketch the graph of $y = -\mathrm{f}(x)$.

b Sketch the graph of $y = 2\mathrm{f}(x)$.



Extend

14 **a** Sketch and label the graph of $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = (x - 1)(x + 1)$.

b On the same axes, sketch and label the graphs of $y = \mathrm{f}(x) - 2$ and $y = \mathrm{f}(x + 2)$.

15 **a** Sketch and label the graph of $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = -(x + 1)(x - 2)$.

b On the same axes, sketch and label the graph of $y = \mathrm{f}\left(-\frac{1}{2}x\right)$.

Straight line graphs

Practice

1 Find the gradient and the y-intercept of the following equations.

a $y = 3x + 5$ **b** $y = -\frac{1}{2}x - 7$

c $2y = 4x - 3$ **d** $x + y = 5$

e $2x - 3y - 7 = 0$ **f** $5x + y - 4 = 0$

> **Hint**
> Rearrange the equations to the form $y = mx + c$

2 Copy and complete the table, giving the equation of the line in the form $y = mx + c$.

Gradient	y-intercept	Equation of the line
5	0	
–3	2	
4	–7	

3 Find, in the form $ax + by + c = 0$ where a, b and c are integers, an equation for each of the lines with the following gradients and y-intercepts.

a gradient $-\frac{1}{2}$, y-intercept –7 **b** gradient 2, y-intercept 0

c gradient $\frac{2}{3}$, y-intercept 4 **d** gradient –1.2, y-intercept –2

4 Write an equation for the line which passes though the point (2, 5) and has gradient 4.

5 Write an equation for the line which passes through the point (6, 3) and has gradient $-\frac{2}{3}$

6 Write an equation for the line passing through each of the following pairs of points.

a (4, 5), (10, 17) **b** (0, 6), (–4, 8)

c (–1, –7), (5, 23) **d** (3, 10), (4, 7)

Extend

7 The equation of a line is $2y + 3x - 6 = 0$.
Write as much information as possible about this line.

Parallel and perpendicular lines

Practice

1 Find the equation of the line parallel to each of the given lines and which passes through each of the given points.

a $y = 3x + 1$ (3, 2) **b** $y = 3 - 2x$ (1, 3)

c $2x + 4y + 3 = 0$ (6, –3) **d** $2y - 3x + 2 = 0$ (8, 20)

2 Find the equation of the line perpendicular to $y = \frac{1}{2}x - 3$ which passes through the point (–5, 3).

> **Hint**
> If $m = \frac{a}{b}$ then the negative reciprocal $-\frac{1}{m} = -\frac{b}{a}$

3 Find the equation of the line perpendicular to each of the given lines and which passes through each of the given points.

a $y = 2x - 6$ (4, 0) **b** $y = -\frac{1}{3}x + \frac{1}{2}$ (2, 13)

c $x - 4y - 4 = 0$ (5, 15) **d** $5y + 2x - 5 = 0$ (6, 7)

4 In each case find an equation for the line passing through the origin which is also perpendicular to the line joining the two points given.

a (4, 3), (–2, –9) **b** (0, 3), (–10, 8)

Extend

5 Work out whether these pairs of lines are parallel, perpendicular or neither.

a $y = 2x + 3$
$y = 2x - 7$

b $y = 3x$
$2x + y - 3 = 0$

c $y = 4x - 3$
$4y + x = 2$

d $3x - y + 5 = 0$
$x + 3y = 1$

e $2x + 5y - 1 = 0$
$y = 2x + 7$

f $2x - y = 6$
$6x - 3y + 3 = 0$

6 The straight line $\mathbf{L_1}$ passes through the points A and B with coordinates (–4, 4) and (2, 1), respectively.

a Find the equation of $\mathbf{L_1}$ in the form $ax + by + c = 0$

The line $\mathbf{L_2}$ is parallel to the line $\mathbf{L_1}$ and passes through the point C with coordinates (–8, 3).

b Find the equation of $\mathbf{L_2}$ in the form $ax + by + c = 0$

The line $\mathbf{L_3}$ is perpendicular to the line $\mathbf{L_1}$ and passes through the origin.

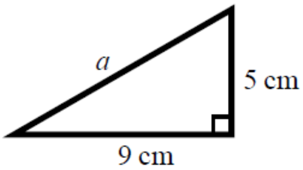
c Find an equation of $\mathbf{L_3}$

Pythagoras’ theorem

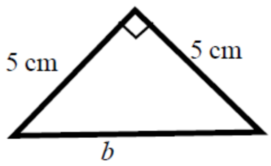
Practice

1 Work out the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

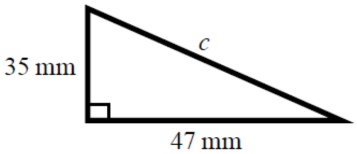
a



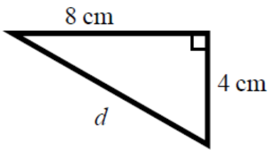
b



c

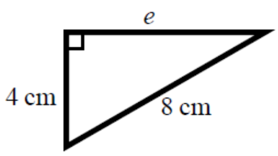


d

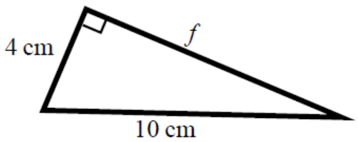


2 Work out the length of the unknown side in each triangle.
Give your answers in surd form.

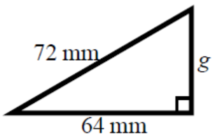
a



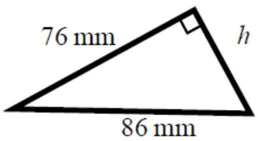
b



c

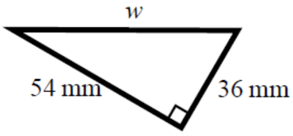


d

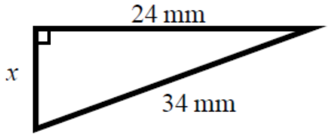


3 Work out the length of the unknown side in each triangle.
Give your answers in surd form.

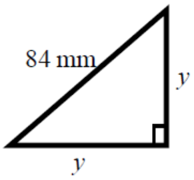
a



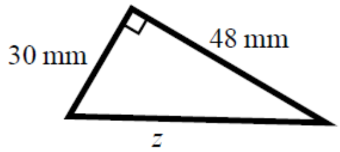
b



c



d



4 A rectangle has length 84 mm and width 45 mm.
Calculate the length of the diagonal of the rectangle.
Give your answer correct to 3 significant figures.

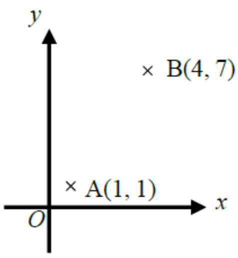
> **Hint**
>
> Draw a sketch of the rectangle.

Extend

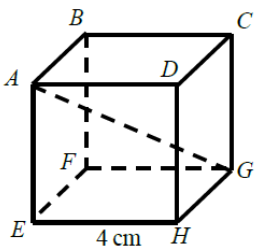
5 A yacht is 40 km due North of a lighthouse.
A rescue boat is 50 km due East of the same lighthouse.
Work out the distance between the yacht and the rescue boat.
Give your answer correct to 3 significant figures.

> **Hint**
>
> Draw a diagram using the information given in the question.

6 Points A and B are shown on the diagram.
Work out the length of the line AB.
Give your answer in surd form.



7 A cube has length 4 cm.
Work out the length of the diagonal *AG*.
Give your answer in surd form.



Proportion

Practice

Hint

Substitute the values given for P and h into the formula to calculate k.

1 Paul gets paid an hourly rate. The amount of pay (£P) is directly proportional to the number of hours (h) he works.
When he works 8 hours he is paid £56.
If Paul works for 11 hours, how much is he paid?

2 x is directly proportional to y.
$x = 35$ when $y = 5$.

a Find a formula for x in terms of y.

b Sketch the graph of the formula.

c Find x when $y = 13$.

d Find y when $x = 63$.

3 Q is directly proportional to the square of Z.
$Q = 48$ when $Z = 4$.

a Find a formula for Q in terms of Z.

b Sketch the graph of the formula.

c Find Q when $Z = 5$.

d Find Z when $Q = 300$.

4 y is directly proportional to the square of x.
$x = 2$ when $y = 10$.

a Find a formula for y in terms of x.

b Sketch the graph of the formula.

c Find x when $y = 90$.

5 B is directly proportional to the square root of C.
$C = 25$ when $B = 10$.

a Find B when $C = 64$.

b Find C when $B = 20$.

6 C is directly proportional to D.
$C = 100$ when $D = 150$.
Find C when $D = 450$.

7 y is directly proportional to x.
$x = 27$ when $y = 9$.
Find x when $y = 3.7$.

8 m is proportional to the cube of n.
$m = 54$ when $n = 3$.
Find n when $m = 250$.

Extend

9 s is inversely proportional to t.

a Given that $s = 2$ when $t = 2$, find a formula for s in terms of t.

b Sketch the graph of the formula.

c Find t when $s = 1$.

10 a is inversely proportional to b.
$a = 5$ when $b = 20$.

a Find a when $b = 50$.

b Find b when $a = 10$.

11 v is inversely proportional to w.
$w = 4$ when $v = 20$.

a Find a formula for v in terms of w.

b Sketch the graph of the formula.

c Find w when $v = 2$.

12 L is inversely proportional to W.
$L = 12$ when $W = 3$.
Find W when $L = 6$.

13 s is inversely proportional to t.
$s = 6$ when $t = 12$.

a Find s when $t = 3$.

b Find t when $s = 18$.

14 y is inversely proportional to x^2.
$y = 4$ when $x = 2$.
Find y when $x = 4$.

15 y is inversely proportional to the square root of x.
$x = 25$ when $y = 1$.
Find x when $y = 5$.

16 a is inversely proportional to b.
$a = 0.05$ when $b = 4$.

a Find a when $b = 2$.

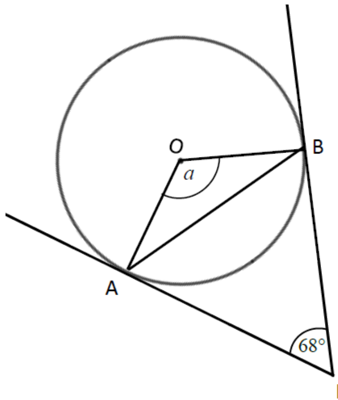
b Find b when $a = 2$.

Circle theorems

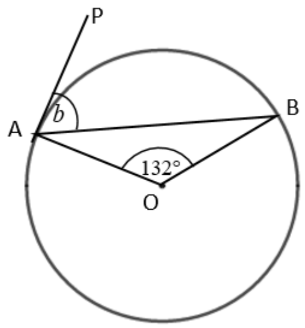
Practice

1 Work out the size of each angle marked with a letter.
Give reasons for your answers.

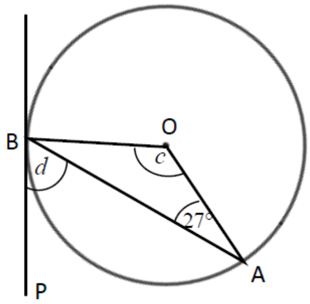
a



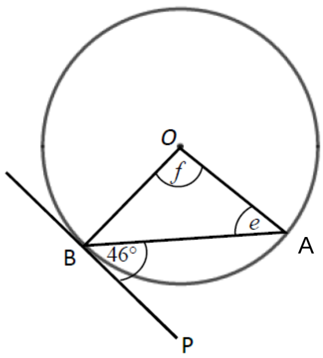
b



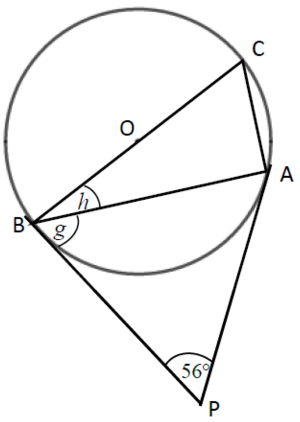
c



d

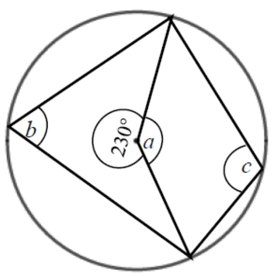


e

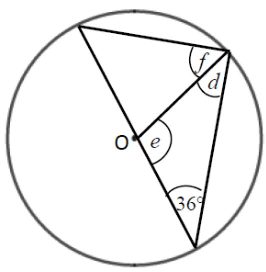


2 Work out the size of each angle marked with a letter.
Give reasons for your answers.

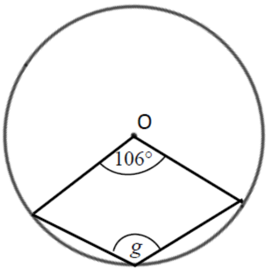
a



b



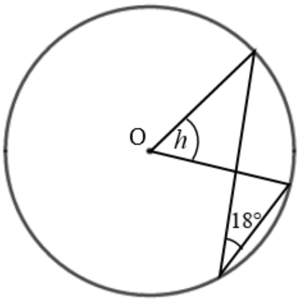
c



Hint

The reflex angle at point O and angle *g* are subtended by the same arc. So the reflex angle is twice the size of angle *g*.

d

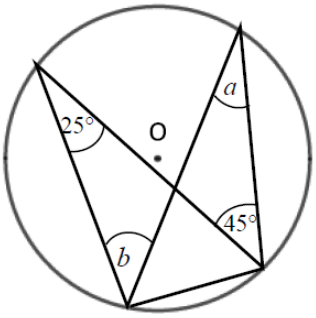


Hint

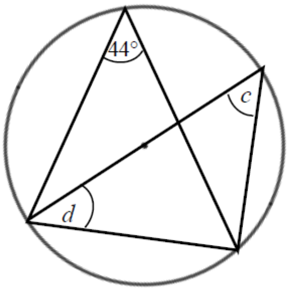
Angle 18° and angle *h* are subtended by the same arc.

3 Work out the size of each angle marked with a letter.
Give reasons for your answers.

a



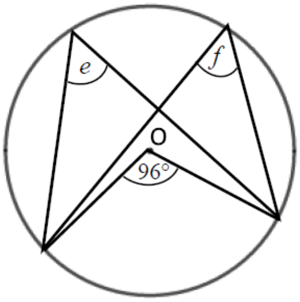
b



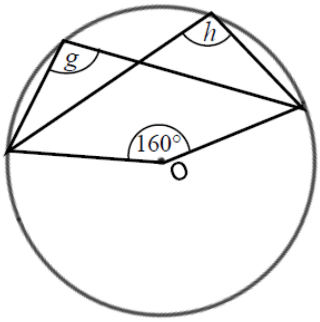
Hint

One of the angles is in a semicircle.

c

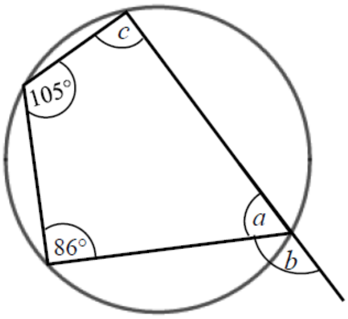


d



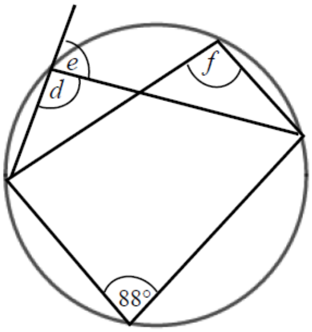
4 Work out the size of each angle marked with a letter.
Give reasons for your answers.

a

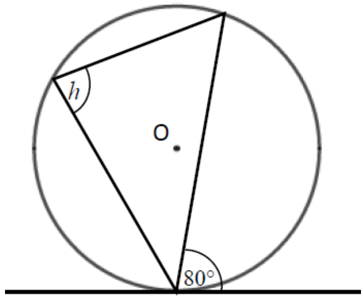


> **Hint**
>
> An exterior angle of a cyclic quadrilateral is equal to the opposite interior angle.

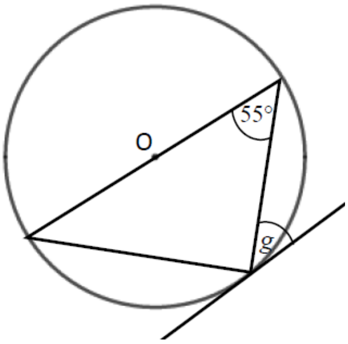
b



c



d



> **Hint**
>
> One of the angles is in a semicircle.

Extend

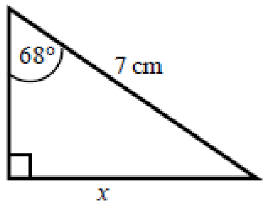
5 Prove the alternate segment theorem.

Trigonometry in right-angled triangles

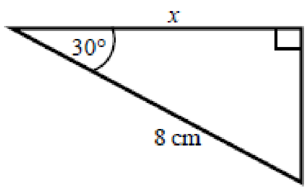
Practice

1 Calculate the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

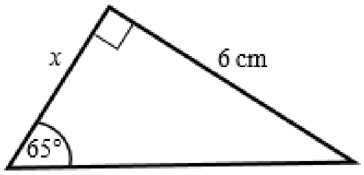
a



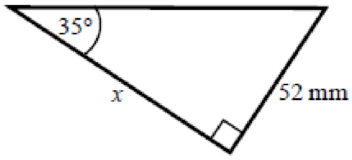
b



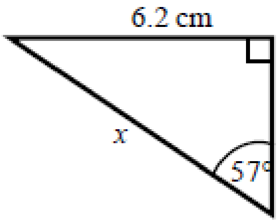
c



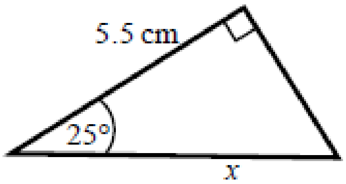
d



e

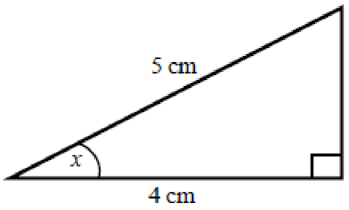


f

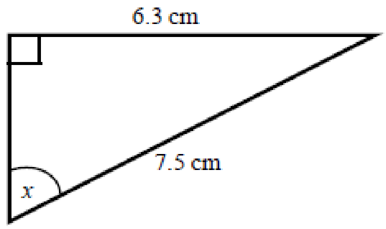


2 Calculate the size of angle x in each triangle.
Give your answers correct to 1 decimal place.

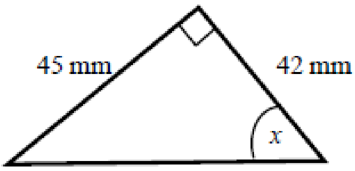
a



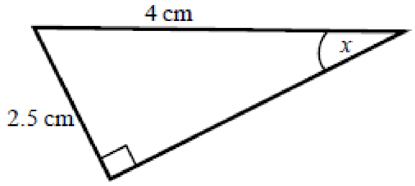
b



c

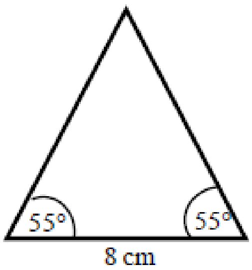


d



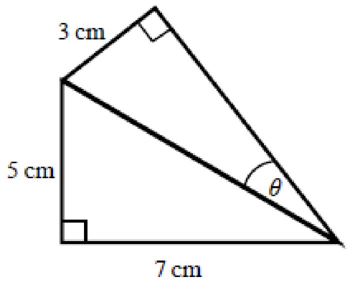
3 Work out the height of the isosceles triangle.
Give your answer correct to 3 significant figures.

> **Hint:**
>
> Split the triangle into two right-angled triangles.



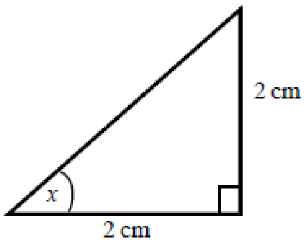
4 Calculate the size of angle θ.
Give your answer correct to 1 decimal place.

> **Hint:**
>
> First work out the length of the common side to both triangles, leaving your answer in surd form.

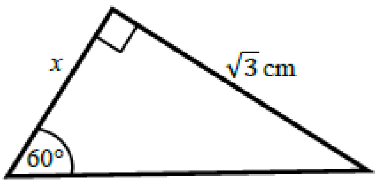


5 Find the exact value of x in each triangle.

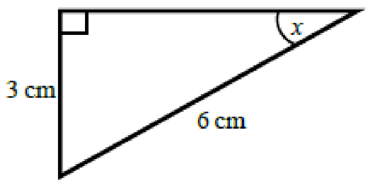
a



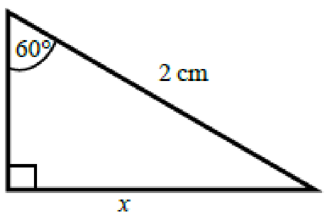
b



c



d

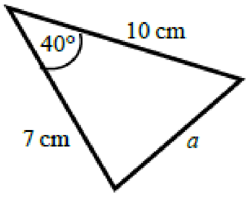


The cosine rule

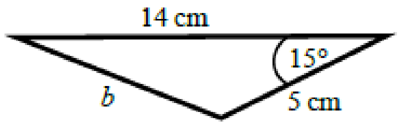
Practice

6 Work out the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

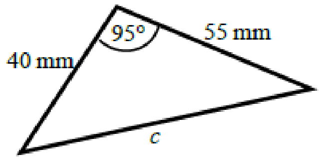
a



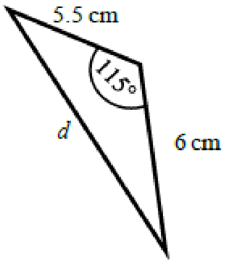
b



c

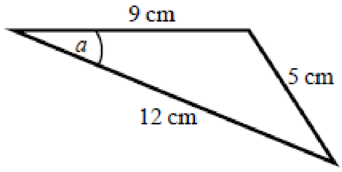


d

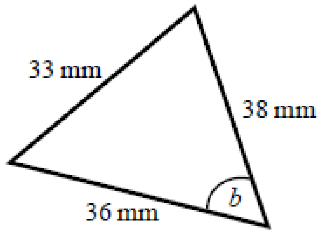


7 Calculate the angles labelled θ in each triangle.
Give your answer correct to 1 decimal place.

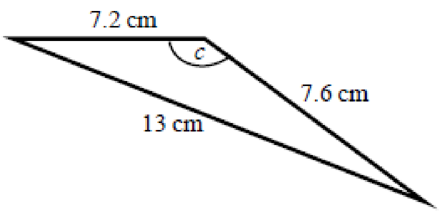
a



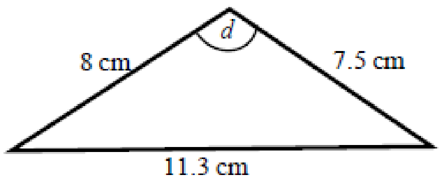
b



c

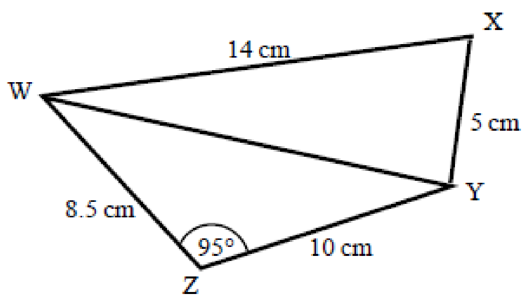


d



8 **a** Work out the length of WY.
Give your answer correct to 3 significant figures.

b Work out the size of angle WXY.
Give your answer correct to 1 decimal place.

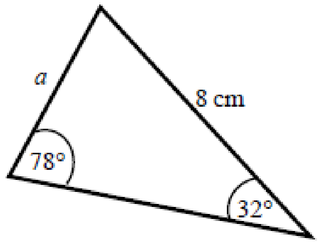


The sine rule

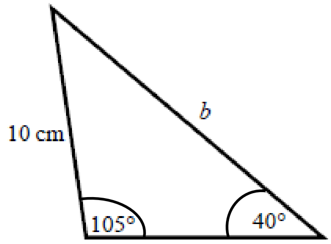
Practice

9 Find the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

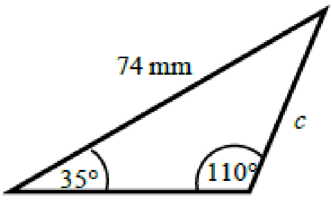
a



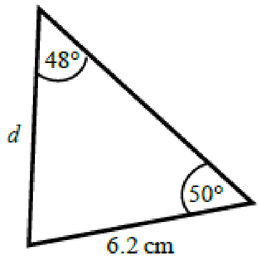
b



c

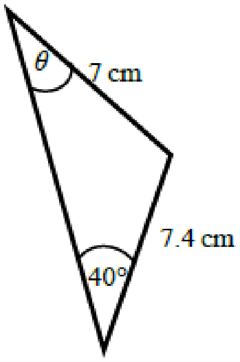


d

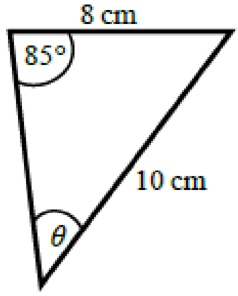


10 Calculate the angles labelled θ in each triangle.
Give your answer correct to 1 decimal place.

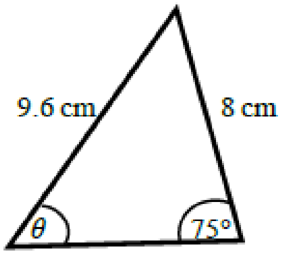
a



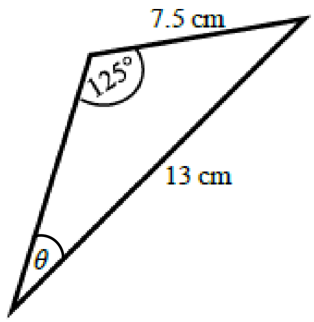
b



c

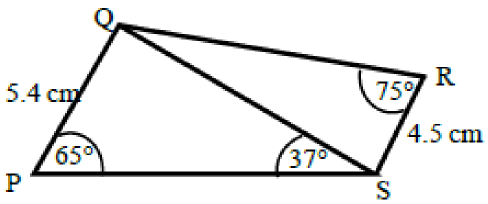


d



11 **a** Work out the length of QS.
Give your answer correct to 3 significant figures.

b Work out the size of angle RQS.
Give your answer correct to 1 decimal place.

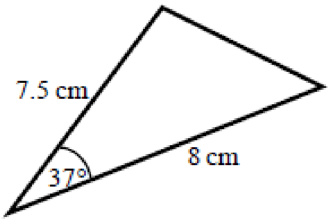


Areas of triangles

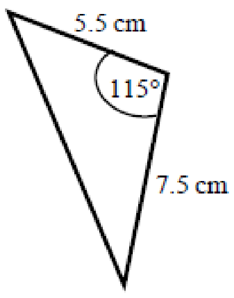
Practice

12 Work out the area of each triangle.
Give your answers correct to 3 significant figures.

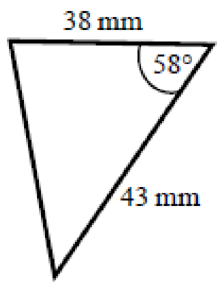
a



b

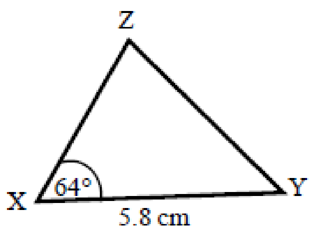


c



13 The area of triangle XYZ is 13.3 cm^2.
Work out the length of XZ.

Hint: Rearrange the formula to make a side the subject.

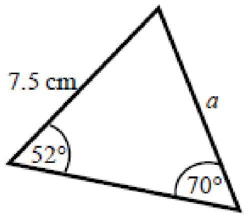


Extend

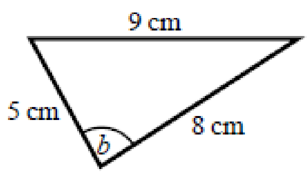
14 Find the size of each lettered angle or side.
Give your answers correct to 3 significant figures.

Hint: For each one, decide whether to use the cosine or sine rule.

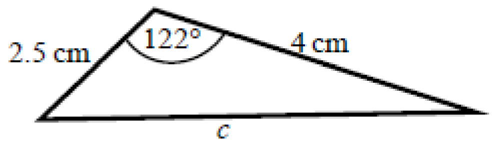
a



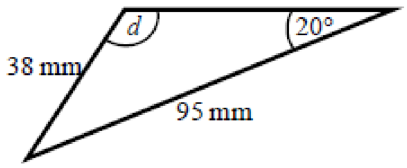
b



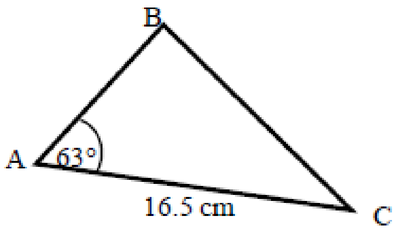
c



d



15 The area of triangle ABC is 86.7 cm^2.
Work out the length of BC.
Give your answer correct to 3 significant figures.



Rearranging equations

Practice

Change the subject of each formula to the letter given in the brackets.

1 $C = \pi d$ [d]

2 $P = 2l + 2w$ [w]

3 $D = \frac{S}{T}$ [T]

4 $p = \frac{q - r}{t}$ [t]

5 $u = at - \frac{1}{2}t$ [t]

6 $V = ax + 4x$ [x]

7 $\frac{y - 7x}{2} = \frac{7 - 2y}{3}$ [y]

8 $x = \frac{2a - 1}{3 - a}$ [a]

9 $x = \frac{b - c}{d}$ [d]

10 $h = \frac{7g - 9}{2 + g}$ [g]

11 $e(9 + x) = 2e + 1$ [e]

12 $y = \frac{2x + 3}{4 - x}$ [x]

13 Make r the subject of the following formulae.

a $A = \pi r^2$ **b** $V = \frac{4}{3}\pi r^3$ **c** $P = \pi r + 2r$ **d** $V = \frac{2}{3}\pi r^2 h$

14 Make x the subject of the following formulae.

a $\frac{xy}{z} = \frac{ab}{cd}$ **b** $\frac{4\pi cx}{d} = \frac{3z}{py^2}$

15 Make sin B the subject of the formula $\frac{a}{\sin A} = \frac{b}{\sin B}$

16 Make cos B the subject of the formula $b^2 = a^2 + c^2 - 2ac \cos B$.

Extend

17 Make x the subject of the following equations.

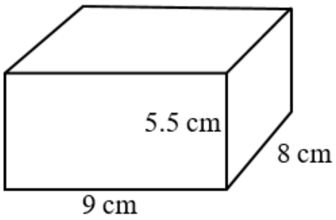
a $\frac{p}{q}(sx + t) = x - 1$ **b** $\frac{p}{q}(ax + 2y) = \frac{3p}{q^2}(x - y)$

Volume and surface area of 3D shapes

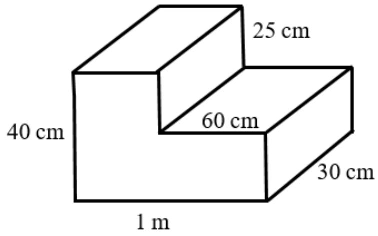
Practice

1 Work out the volume of each solid.
Leave your answers in terms of π where appropriate.

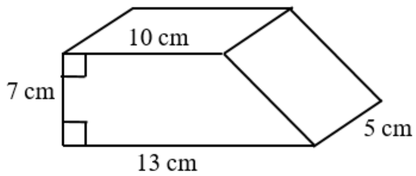
a



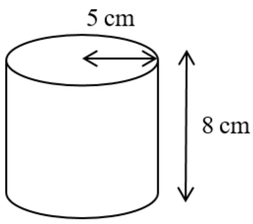
b



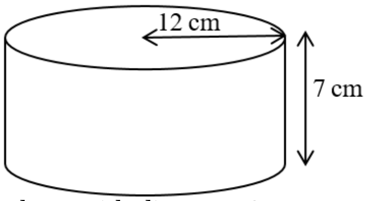
c



d



e

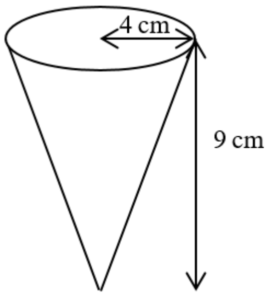


f a sphere with radius 7 cm

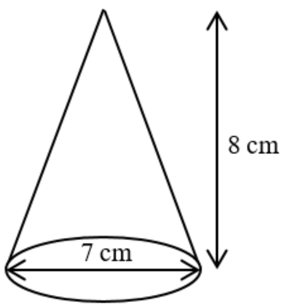
g a sphere with diameter 9 cm

h a hemisphere with radius 3 cm

i

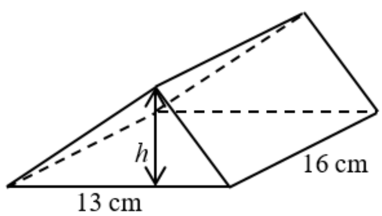


j



2 A cuboid has width 9.5 cm, height 8 cm and volume 1292 cm^3.
Work out its length.

3 The triangular prism has volume 1768 cm^3.
Work out its height.



Extend

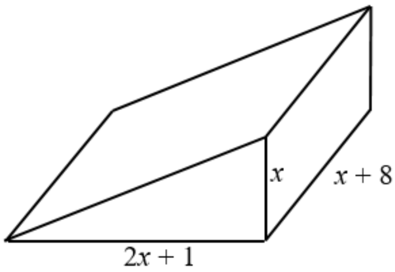
4 The diagram shows a solid triangular prism.

All the measurements are in centimetres.

The volume of the prism is V cm^3.

Find a formula for V in terms of x.

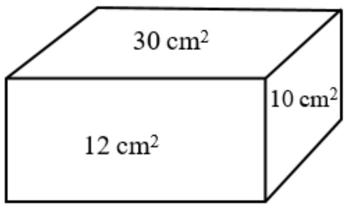
Give your answer in simplified form.



5 The diagram shows the area of each of three faces of a cuboid.

The length of each edge of the cuboid is a whole number of centimetres.

Work out the volume of the cuboid.



6 The diagram shows a large catering size tin of beans in the shape of a cylinder.

The tin has a radius of 8 cm and a height of 15 cm.

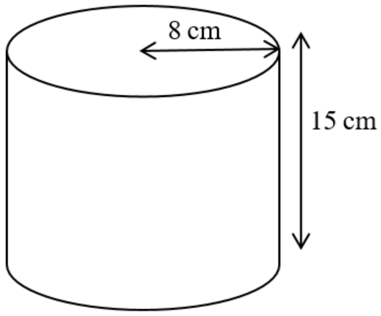
A company wants to make a new size of tin.

The new tin will have a radius of 6.7 cm.

It will have the same volume as the large tin.

Calculate the height of the new tin.

Give your answer correct to one decimal place.



7 The diagram shows a sphere and a solid cylinder.

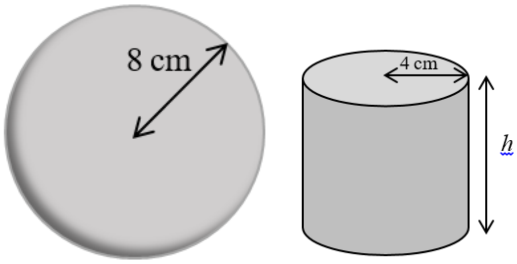
The sphere has radius 8 cm.

The solid cylinder has a base radius of 4 cm and a height of h cm.

The total surface area of the cylinder is half the total surface area of the sphere.

Work out the ratio of the volume of the sphere to the volume of the cylinder.

Give your answer in its simplest form.

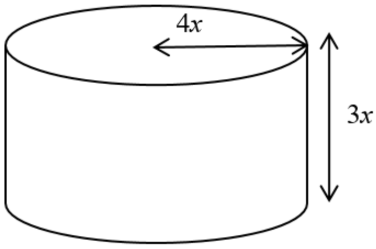


8 The diagram shows a solid metal cylinder.

The cylinder has base radius $4x$ and height $3x$.

The cylinder is melted down and made into a sphere of radius r.

Find an expression for r in terms of x.



Area under a graph

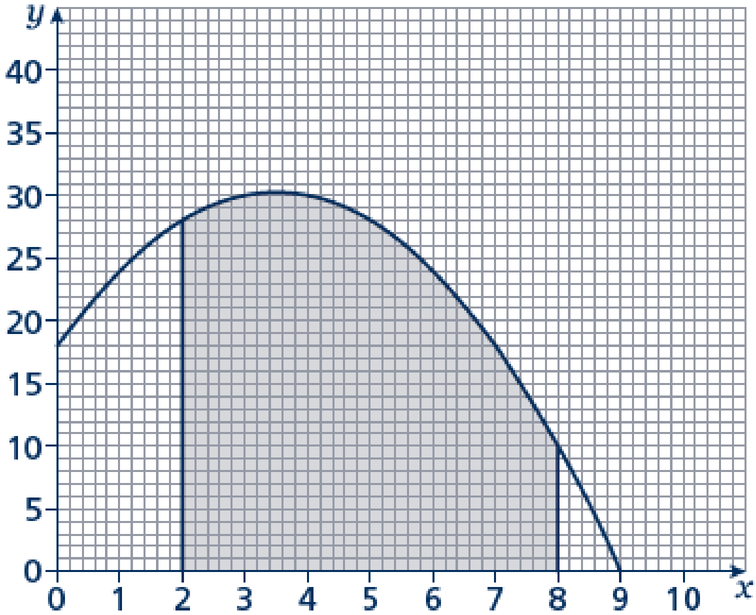
Practice

Hint:

For a full answer, remember to include 'units^2'.

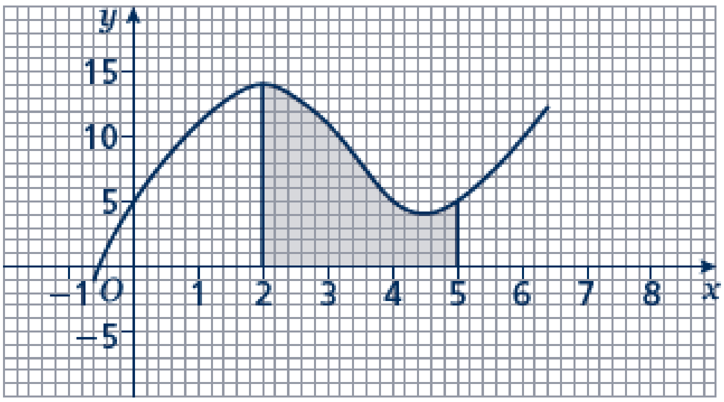
1 Estimate the area of the region between the curve $y = (5 - x)(x + 2)$ and the x-axis from $x = 1$ to $x = 5$.
Use four strips of width 1 unit.

2 Estimate the shaded area shown on the axes.
Use six strips of width 1 unit.



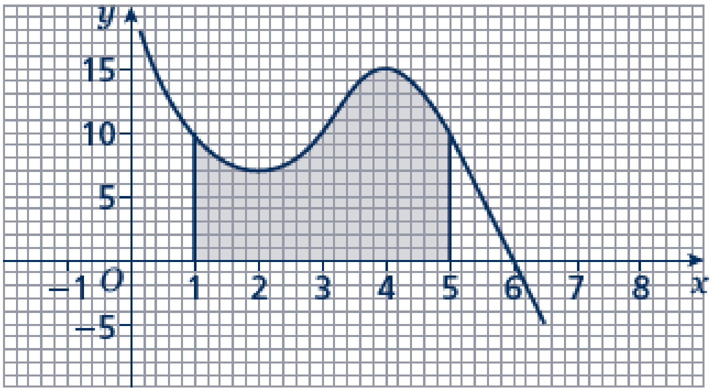
3 Estimate the area of the region between the curve $y = x^2 - 8x + 18$ and the x-axis from $x = 2$ to $x = 6$.
Use four strips of width 1 unit.

4 Estimate the shaded area.
Use six strips of width $\frac{1}{2}$ unit.



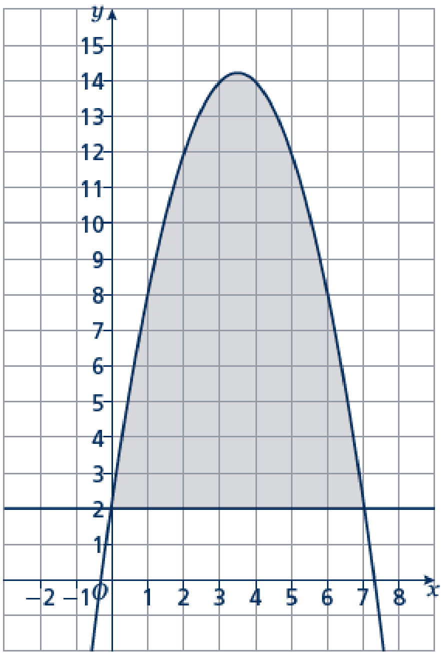
5 Estimate the area of the region between the curve $y = -x^2 - 4x + 5$ and the x-axis from $x = -5$ to $x = 1$.
Use six strips of width 1 unit.

6 Estimate the shaded area.
Use four strips of equal width.



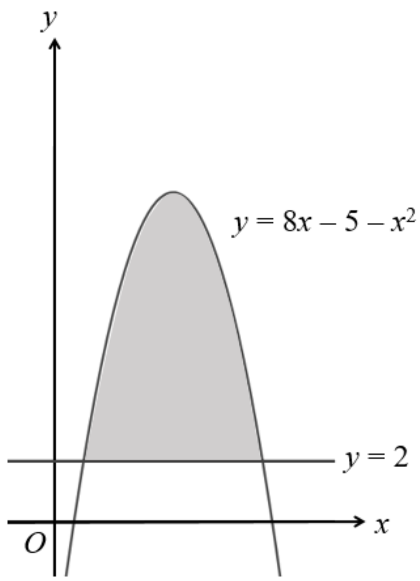
7 Estimate the area of the region between the curve $y = -x^2 + 2x + 15$ and the x-axis from $x = 2$ to $x = 5$.
Use six strips of equal width.

8 Estimate the shaded area.
Use seven strips of equal width.

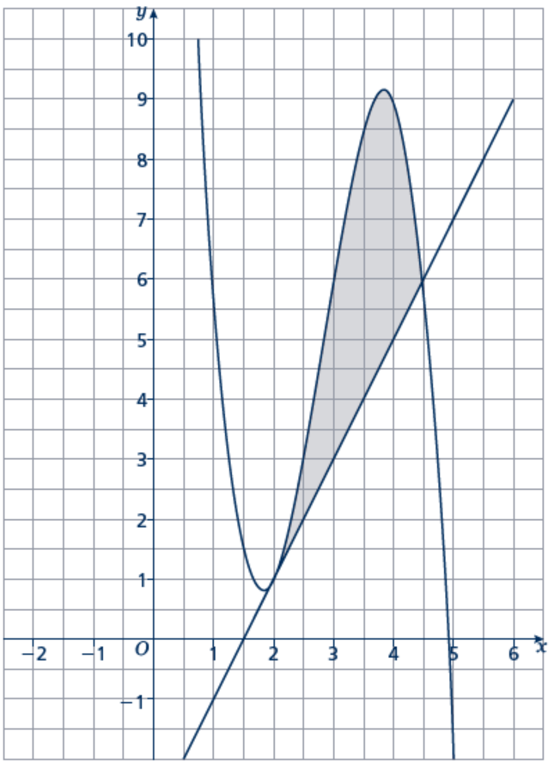


Extend

9 The curve $y = 8x - 5 - x^2$ and the line $y = 2$ are shown in the sketch.
Estimate the shaded area using six strips of equal width.



10 Estimate the shaded area using five strips of equal width.



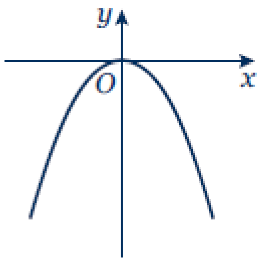
Section C

Answers

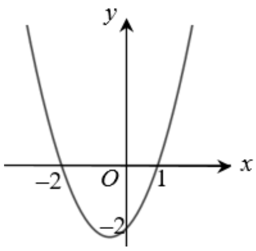
Sketching quadratic graphs

Answers

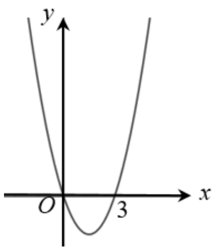
1



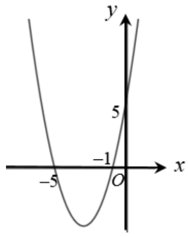
2 **a**



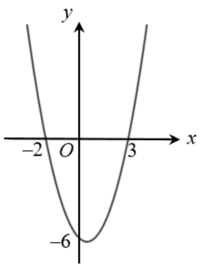
b



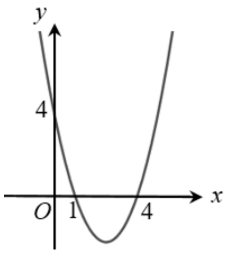
c



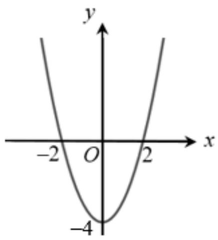
3 **a**



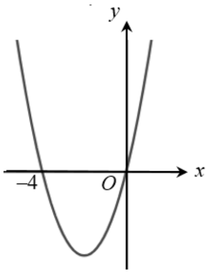
b



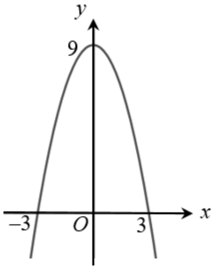
c



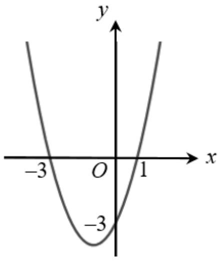
d



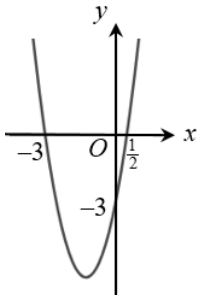
e



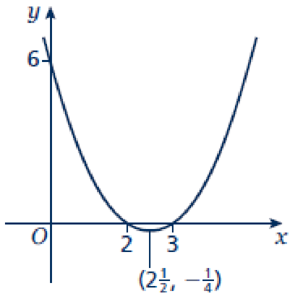
f



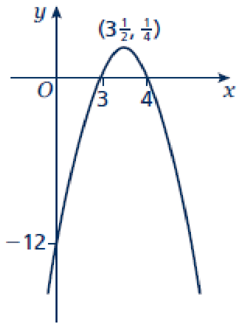
4



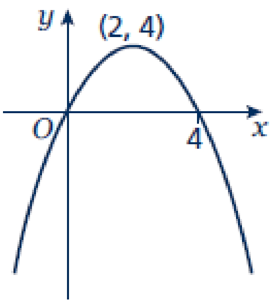
5 **a**



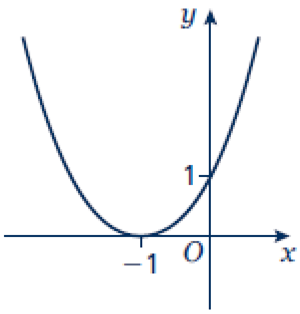
b



c



6



Line of symmetry at $x = -1$.

Solving simultaneous equations graphically

Answers

1 **a** $x = 2$, $y = 5$
b $x = 2$, $y = -3$
c $x = -0.5$, $y = 2.5$

2 **a** $x = -2$, $y = 2$
b $x = 0.5$, $y = 0.5$
c $x = -1$, $y = -2$

3 **a** $x = 1$, $y = 0$ and $x = 4$, $y = 3$
b $x = -2$, $y = 7$ and $x = 2$, $y = -5$
c $x = -2$, $y = 5$ and $x = -1$, $y = 4$

4 $x = -3$, $y = 4$ and $x = 4$, $y = -3$

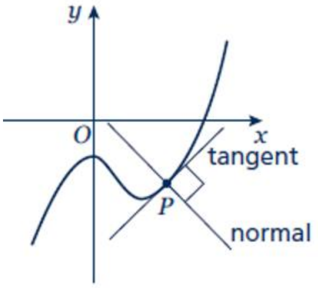
5 **a** **i** $x = 2.5$, $y = -2$ and $x = -0.5$, $y = 4$
ii $x = 2.41$, $y = -1.83$ and $x = -0.41$, $y = 3.83$
b Solving algebraically gives the more accurate solutions as the solutions from the graph are only estimates, based on the accuracy of your graph.

Sketching cubic and reciprocal graphs

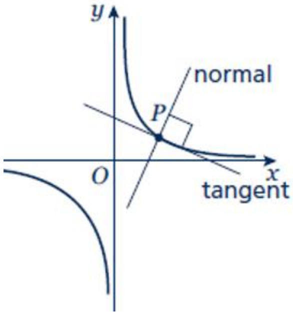
Answers

1 **a** i – C
ii – E
iii – B
iv – A
v – F
vi – D

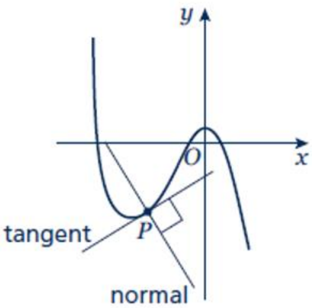
b **ii**



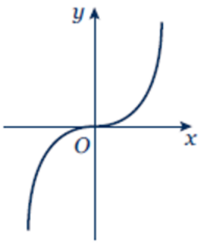
iv



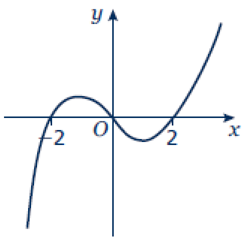
vi



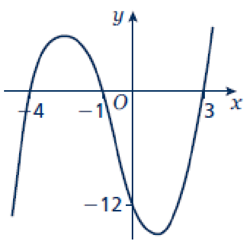
2



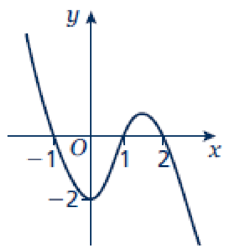
3



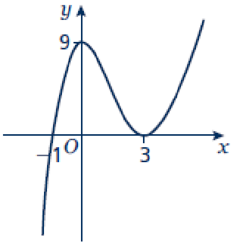
4



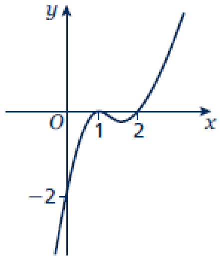
5



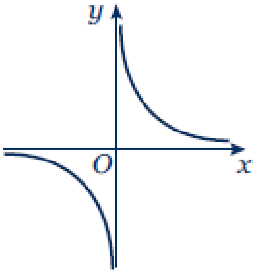
6



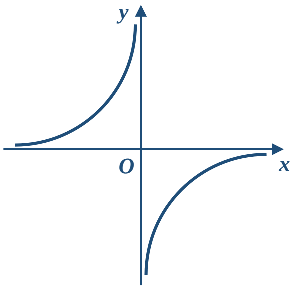
7



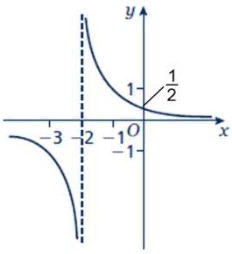
8



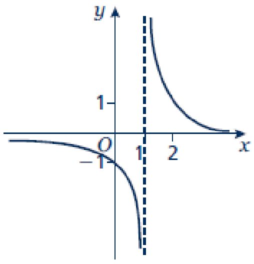
9



10



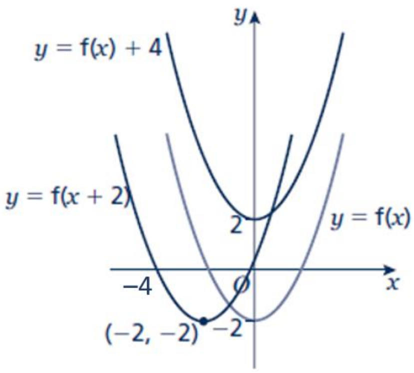
11



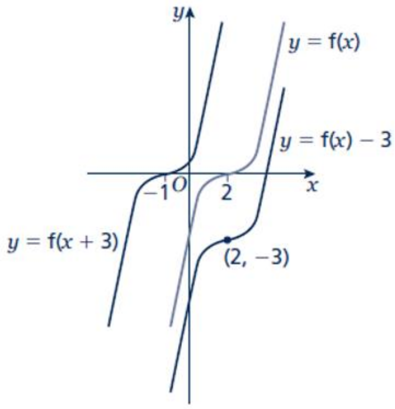
Translating & Stretching graphs

Answers

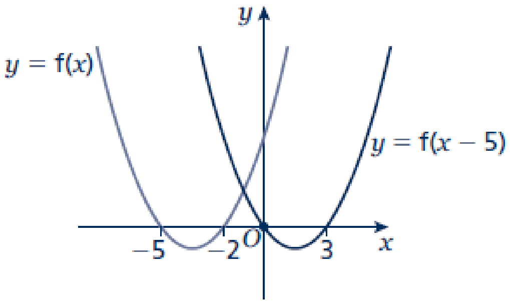
1



2



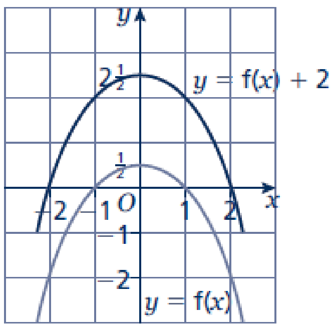
3



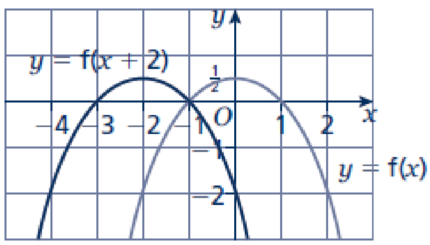
4 C_1: $y = f(x - 90°)$
C_2: $y = f(x) - 2$

5 C_1: $y = f(x - 5)$
C_2: $y = f(x) - 3$

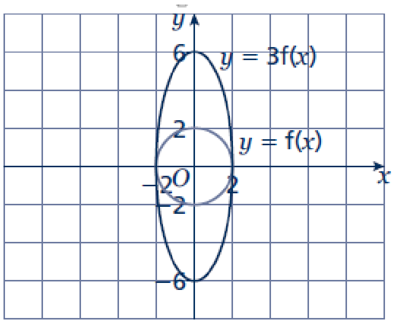
6 **a**



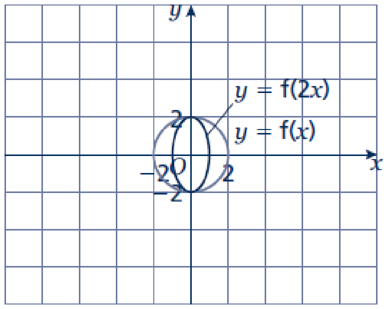
b



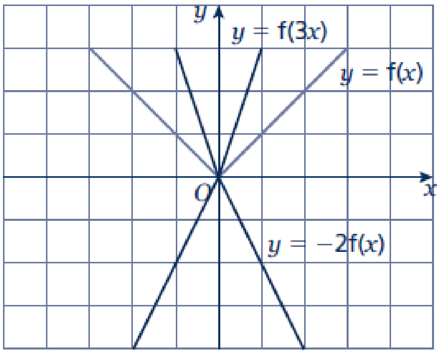
7 **a**



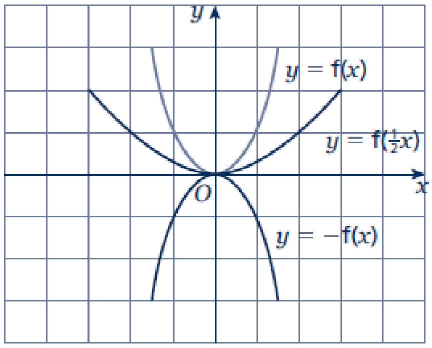
b



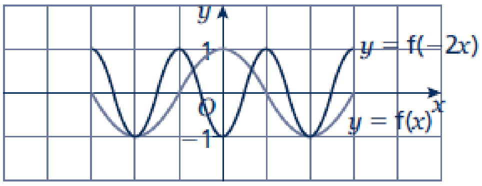
8



9



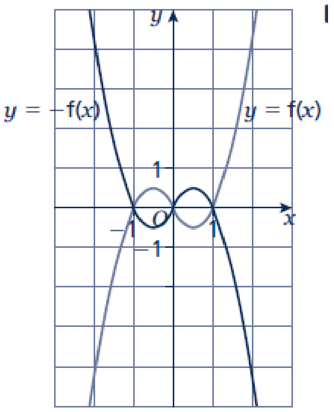
10



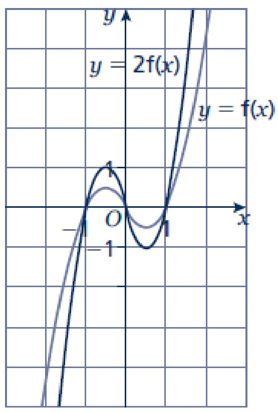
11 $y = \mathrm{f}(2x)$

12 $y = -2\mathrm{f}(2x)$ or $y = 2\mathrm{f}(-2x)$

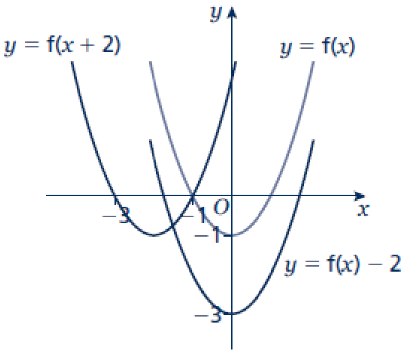
13 **a**



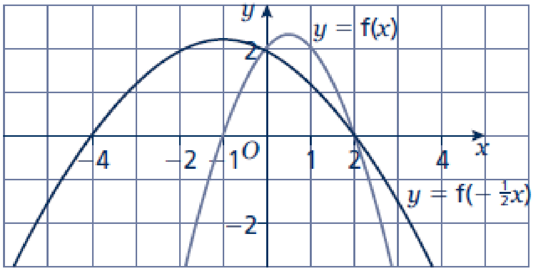
b



14



15



Straight line graphs

Answers

1 **a** $m = 3, c = 5$ **b** $m = -\frac{1}{2}, c = -7$

c $m = 2, c = -\frac{3}{2}$ **d** $m = -1, c = 5$

e $m = \frac{2}{3}, c = -\frac{7}{3}$ or $-2\frac{1}{3}$ **f** $m = -5, c = 4$

2

Gradient	*y*-intercept	Equation of the line
5	0	$y = 5x$
–3	2	$y = -3x + 2$
4	–7	$y = 4x - 7$

3 **a** $x + 2y + 14 = 0$ **b** $2x - y = 0$

c $2x - 3y + 12 = 0$ **d** $6x + 5y + 10 = 0$

4 $y = 4x - 3$

5 $y = -\frac{2}{3}x + 7$

6 **a** $y = 2x - 3$ **b** $y = -\frac{1}{2}x + 6$

c $y = 5x - 2$ **d** $y = -3x + 19$

7 $y = -\frac{3}{2}x + 3$, the gradient is $-\frac{3}{2}$ and the *y*-intercept is 3.

The line intercepts the axes at (0, 3) and (2, 0).

Students may sketch the line or give coordinates that lie on the line such as $\left(1, \frac{3}{2}\right)$ or $(4, -3)$.

Parallel and perpendicular lines

Answers

1 **a** $y = 3x - 7$ **b** $y = -2x + 5$

c $y = -\frac{1}{2}x$ **d** $y = \frac{3}{2}x + 8$

2 $y = -2x - 7$

3 **a** $y = -\frac{1}{2}x + 2$ **b** $y = 3x + 7$

c $y = -4x + 35$ **d** $y = \frac{5}{2}x - 8$

4 **a** $y = -\frac{1}{2}x$ **b** $y = 2x$

5 **a** Parallel **b** Neither **c** Perpendicular

d Perpendicular **e** Neither **f** Parallel

6 **a** $x + 2y - 4 = 0$ **b** $x + 2y + 2 = 0$ **c** $y = 2x$

Pythagoras' theorem

Answers

1	**a**	10.3 cm	**b**	7.07 cm
	c	58.6 mm	**d**	8.94 cm
2	**a**	$4\sqrt{3}$ cm	**b**	$2\sqrt{21}$ cm
	c	$8\sqrt{17}$ mm	**d**	$18\sqrt{5}$ mm
3	**a**	$18\sqrt{13}$ mm	**b**	$2\sqrt{145}$ mm
	c	$42\sqrt{2}$ mm	**d**	$6\sqrt{89}$ mm

4 95.3 mm

5 64.0 km

6 $3\sqrt{5}$ units

7 $4\sqrt{3}$ cm

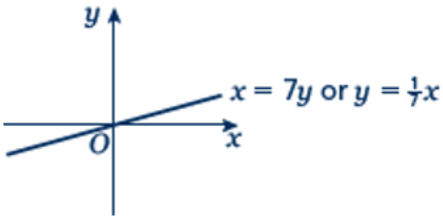
Proportion

Answers

1 £77

2 **a** $x = 7y$

b

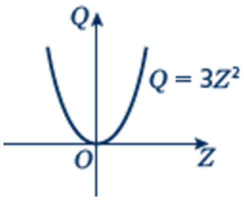


c 91

d 9

3 **a** $Q = 3Z^2$

b

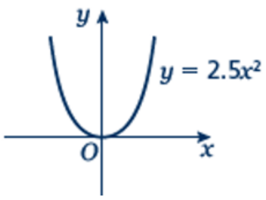


c 75

d ±10

4 **a** $y = 2.5x^2$

b



c ±6

5 **a** 16

b 100

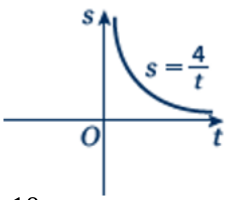
6 300

7 11.1

8 5

9 **a** $s = \dfrac{4}{t}$

b



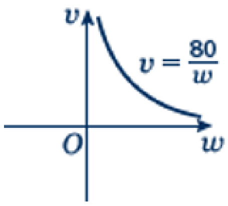
c 4

10 **a** 2

b 10

11 **a** $v = \dfrac{80}{w}$

b



c 40

12 6

13 **a** 24 **b** 4

14 1

15 1

16 **a** 0.1 **b** 0.1

Circle theorems

Answers

1 **a** $a = 112°$, angle OAP = angle OBP = 90° and angles in a quadrilateral total 360°.

b $b = 66°$, triangle OAB is isosceles, Angle OAP = 90° as AP is tangent to the circle.

c $c = 126°$, triangle OAB is isosceles.
$d = 63°$, Angle OBP = 90° as BP is tangent to the circle.

d $e = 44°$, the triangle is isosceles, so angles *e* and angle OBA are equal. The angle OBP = 90° as BP is tangent to the circle.
$f = 92°$, the triangle is isosceles.

e $g = 62°$, triangle ABP is isosceles as AP and BP are both tangents to the circle.
$h = 28°$, the angle OBP = 90°.

2 **a** $a = 130°$, angles in a full turn total 360°.
$b = 65°$, the angle at the centre of a circle is twice the angle at the circumference.
$c = 115°$, opposite angles in a cyclic quadrilateral total 180°.

b $d = 36°$, isosceles triangle.
$e = 108°$, angles in a triangle total 180°.
$f = 54°$, angle in a semicircle is 90°.

c $g = 127°$, angles at a full turn total 360°, the angle at the centre of a circle is twice the angle at the circumference.

d $h = 36°$, the angle at the centre of a circle is twice the angle at the circumference.

3 **a** $a = 25°$, angles in the same segment are equal.
$b = 45°$, angles in the same segment are equal.

b $c = 44°$, angles in the same segment are equal.
$d = 46°$, the angle in a semicircle is 90° and the angles in a triangle total 180°.

c $e = 48°$, the angle at the centre of a circle is twice the angle at the circumference.
$f = 48°$, angles in the same segment are equal.

d $g = 100°$, angles at a full turn total 360°, the angle at the centre of a circle is twice the angle at the circumference.
$h = 100°$, angles in the same segment are equal.

4 **a** $a = 75°$, opposite angles in a cyclic quadrilateral total 180°.
$b = 105°$, angles on a straight line total 180°.
$c = 94°$, opposite angles in a cyclic quadrilateral total 180°.

b $d = 92°$, opposite angles in a cyclic quadrilateral total 180°.
$e = 88°$, angles on a straight line total 180°.
$f = 92°$, angles in the same segment are equal.

c $h = 80°$, alternate segment theorem.

d $g = 35°$, alternate segment theorem and the angle in a semicircle is 90°.

5 Angle BAT = x.

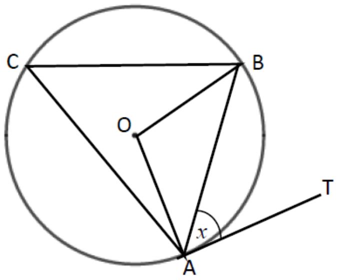
Angle OAB = $90° - x$ because the angle between the tangent and the radius is 90°.

OA = OB because radii are equal.

Angle OAB = angle OBA because the base of isosceles triangles are equal.

Angle AOB = $180° - (90° - x) - (90° - x) = 2x$ because angles in a triangle total 180°.

Angle ACB = $2x \div 2 = x$ because the angle at the centre is twice the angle at the circumference.



Trigonometry in right-angled triangles - The cosine rule, The sine rule & Areas of triangles

Answers

1	**a**	6.49 cm	**b**	6.93 cm	**c**	2.80 cm		
	d	74.3 mm	**e**	7.39 cm	**f**	6.07 cm		
2	**a**	36.9°	**b**	57.1°	**c**	47.0°	**d**	38.7°
3	5.71 cm							
4	20.4°							
5	**a**	45°	**b**	1 cm	**c**	30°	**d**	$\sqrt{3}$ cm
6	**a**	6.46 cm	**b**	9.26 cm	**c**	70.8 mm	**d**	9.70 cm
7	**a**	22.2°	**b**	52.9°	**c**	122.9°	**d**	93.6°
8	**a**	13.7 cm	**b**	76.0°				
9	**a**	4.33 cm	**b**	15.0 cm	**c**	45.2 mm	**d**	6.39 cm
10	**a**	42.8°	**b**	52.8°	**c**	53.6°	**d**	28.2°
11	**a**	8.13 cm	**b**	32.3°				
12	**a**	18.1 cm^2	**b**	18.7 cm^2	**c**	693 mm^2		
13	5.10 cm							
14	**a**	6.29 cm	**b**	84.3°	**c**	5.73 cm	**d**	58.8°
15	15.3 cm							

Rearranging equations

Answers

1 $d = \frac{C}{\pi}$

2 $w = \frac{P - 2l}{2}$

3 $T = \frac{S}{D}$

4 $t = \frac{q - r}{p}$

5 $t = \frac{2u}{2a - 1}$

6 $x = \frac{V}{a + 4}$

7 $y = 2 + 3x$

8 $a = \frac{3x + 1}{x + 2}$

9 $d = \frac{b - c}{x}$

10 $g = \frac{2h + 9}{7 - h}$

11 $e = \frac{1}{x + 7}$

12 $x = \frac{4y - 3}{2 + y}$

13 **a** $r = \sqrt{\frac{A}{\pi}}$

b $r = \sqrt[3]{\frac{3V}{4\pi}}$

c $r = \frac{P}{\pi + 2}$

d $r = \sqrt{\frac{3V}{2\pi h}}$

14 **a** $x = \frac{abz}{cdy}$

b $x = \frac{3dz}{4\pi cpy^2}$

15 $\sin B = \frac{b \sin A}{a}$

16 $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

17 **a** $x = \frac{q + pt}{q - ps}$

b $x = \frac{3py + 2pqy}{3p - apq} = \frac{y(3 + 2q)}{3 - aq}$

Volume and surface area of 3D shapes

Answers

1 **a** $V = 396\ \text{cm}^3$ **b** $V = 75\,000\ \text{cm}^3$

c $V = 402.5\ \text{cm}^3$ **d** $V = 200\pi\ \text{cm}^3$

e $V = 1008\pi\ \text{cm}^3$ **f** $V = \frac{1372}{3}\pi\ \text{cm}^3$

g $V = 121.5\pi\ \text{cm}^3$ **h** $V = 18\pi\ \text{cm}^3$

i $V = 48\pi\ \text{cm}^3$ **j** $V = \frac{98}{3}\pi\ \text{cm}^3$

2 17 cm

3 17 cm

4 $V = x^3 + \frac{17}{2}x^2 + 4x$

5 $60\ \text{cm}^3$

6 21.4 cm

7 32 : 9

8 $r = \sqrt[3]{36x}$

Area under a graph

Answers

1 34 units^2

2 149 units^2

3 14 units^2

4 $25\frac{1}{4}$ units^2

5 35 units^2

6 42 units^2

7 $26\frac{7}{8}$ units^2

8 56 units^2

9 35 units^2

10 $6\frac{1}{4}$ units^2